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THE CONNECTED COMPONENT OF THE IDELE CLASS GROUP OF AN ALGEBRAIC NUMBER FIELD

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We shall give another proof of Weil's theorem of the structure of the connected component of the idèle class group of an algebraic number field. Our proof is different from Artin's.

Let Q be the rational number field and k be an algebraic number field of finite degree over Q. We denote by C_k the idèle class group of k and D_k the connected component of unity of C_k. Let T denote the multiplicative group of all complex numbers of absolute value 1 with compact topology, R the additive group of the real numbers with usual topology, and S the Solenoid with compact topology.

Weil ([**3**]) has shown that D_k is isomorphic to $T^{r_2} \times R \times S^r$, by determining the structure of the dual D_k^*. Artin ([**1**]) has exhibited a system of representatives of idèle classes and given a different proof. In this paper we shall give another proof of the above Weil's theorem.

1. Let k be an algebraic number field which has r_1 real infinite primes and r_2 complex infinite primes. As usual we put $r = r_1 + r_2 - 1$. Let I_k be the idèle group of k, C_k the idèle class group of k and D_k the connected component of unity of C_k. An idèle will be denoted by $(a_v) = (a_\mathfrak{p}, a_\lambda)$, where v runs all primes of k, $\mathfrak{p}$ all finite primes and λ all infinite primes of k ($\lambda = 1,\ldots,r_1 + r_2$). We shall agree that λ ($1 \le \lambda \le r_1$) is real and λ ($r_1 + 1 \le \lambda \le r_1 + r_2$) is complex. Let us denote by σ_λ the embedding of k into the complex number field attached to an infinite prime λ. Then σ_λ with $1 \le \lambda \le r_1$ is a real embedding and σ_λ with $r_1 + 1 \le \lambda \le r_1 + r_2$ a complex one.

For any topological group G, G^* denotes the character group of G. If χ is a character of C_k, i.e., a continuous homomorphism of C_k into T, we can regard it as a character of I_k which is trivial on principal idèles. If we restrict χ to the infinite part $R^{\times r_1} C^{\times r_2}$ of I_k, χ can be written as follows:

$$\chi((a_\lambda)) = \prod_{\lambda=1}^{r_1+r_2} \left(\frac{a_\lambda}{|a_\lambda|} \right)^{f_\lambda} |a_\lambda|^{\sqrt{-1}\,\varphi_\lambda}, \qquad (a_\lambda) \in R^{\times r_1} C^{\times r_2},$$

where $f_\lambda \in Z$ (the rational integers), $\varphi_\lambda \in R$ ($\lambda = 1,\ldots,r_1 + r_2$), and $f_1,\ldots,f_{r_1} = 0$ or 1. Such f_λ and φ_λ ($\lambda = 1,\ldots,r_1 + r_2$) are uniquely determined, so we say that χ is of type $(f_\lambda, \varphi_\lambda)$.