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CLIFFORD'S THEOREM FOR ALGEBRAIC GROUPS AND LIE ALGEBRAS

JOHN W. BALLARD

The standard results for comparing the irreducible representations of a group to those of a normal subgroup were obtained by A. H. Clifford. The object of this paper is to discuss a variation of these results in which the group is assumed to be an affine algebraic group and the role of the normal subgroup is played by the Lie algebra.

1. Introduction. Let G be an affine algebraic group over an algebraically closed field of positive characteristic. Let V be an irreducible rational representation space for G. We show that when viewed as a representation space for the Lie algebra $\mathfrak{g}$, V decomposes as the direct sum of isomorphic irreducible representations. This is the analog of the first two results in [**4**]. Let W be an irreducible $\mathfrak{g}$-subspace of V. We show that V factors as the tensor product of two rational projective representations of G, one of which is induced by W while the other is a representation of the quotient group $G/\mathfrak{g}$. Since the field has positive characteristic it follows that $V = W \otimes U$, where U is the Frobenius power of a projective representation of G. This is the analog of Clifford's Theorem 3. We assume that the Lie algebra has no non-trivial one dimensional restricted representations and show that this factorization may be continued to express V as the tensor product of irreducible representations of $\mathfrak{g}$ and their Frobenius powers. The Curtis-Steinberg decomposition [**3**] for the irreducible representations of a simply connected semisimple group then follows as a corollary.

In the course of the discussion we construct a Schur representation group G^s of G relative to its Lie algebra and a relative Schur multiplier. Essentially G^s is the smallest covering group of G which linearizes the irreducible representations of $\mathfrak{g}$. Our identification of G^s as the simply connected covering group of G answers a question originally posed by Curtis [**6**, p. 325] in the context of Chevalley groups. We identify the relative Schur multiplier with the Picard group of the algebraic variety G. This amounts to showing that the Picard group is generated by the irreducible representations of the Lie algebra which do not arise as the differential of a representation for the group.