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INTEGRAL INVARIANTS OF FUNCTIONS AND L^p ISOMETRIES ON GROUPS

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For $p \in (0, \infty)$ and not an even integer it is proved that every isometric multiplier on an invariant subspace of $L^p(G)$ is a translation operator.

1. Introduction. Let f and g be real-valued measurable functions on $\mathbf{R}$ which satisfy

$$\int_{\mathbf{R}} \left| \sum \alpha_j f(t_j + x) \right|^p dx = \int_{\mathbf{R}} \left| \sum \alpha_j g(t_j + x) \right|^p dx < \infty$$

for arbitrary finite sets of real numbers $\{\alpha_j\}$ and $\{t_j\}$. In [3], M. Kanter showed that if $p \in (0, \infty)$ is not an even integer then for some $\varepsilon = \pm 1$ and some $t_0 \in \mathbf{R}$, $g(x) = \varepsilon f(t_0 + x)$ a.e. When rephrased in the language of multipliers, Kanter's theorem becomes: let F be the closed linear span in $L^p(\mathbf{R})$ of the translates of f. Suppose $p \in (0, \infty)$ is not an even integer and that R: $F \to L^p(\mathbf{R})$ is an isometry which commutes with translations. Then for some $\varepsilon = \pm 1$ and some $t_0 \in \mathbf{R}$

$$Rf(x) = \varepsilon f(t_0 + x).$$

A related theorem was proved by R. S. Strichartz [8] in the case of a locally compact group. Namely, Strichartz showed that if $p \in [1, \infty)$ and $p \neq 2$ then each invertible isometric multiplier on $L^p(G)$ is a translation operator. Since the space F in Kanter's theorem need not equal $L^p(\mathbf{R})$ it is clear that Kanter's theorem does not follow from that of Strichartz. Also since Strichartz's theorem only requires that $p \neq 2$ and Strichartz's group is arbitrary it is clear that his results do not follow from those of Kanter.

The main result in this paper is an extension of Kanter's theorem to an arbitrary locally compact group G. The restriction that p is not an even integer is still needed but we will see that the proof also contains new information for $p \neq 2$.

Concerning the restriction on p, Strichartz's result is known to be false if $p = 2$. Katznelson [4] showed that Kanter's theorem fails if p is an even integer. Precisely what does happen for $p = 2n \geq 4$ is not yet understood but is related to the work of R. L. Adler and A. G. Konheim on higher order autocorrelation functions on abelian groups [1]. In §5 we