PACIFIC JOURNAL OF MATHEMATICS
Vol. 106, No. 2, 1983

AMPLENESS IN COMPLEX HOMOGENEOUS SPACES AND A SECOND LEFSCHETZ THEOREM

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This paper investigates how ampleness of the normal bundle of a smooth subvariety Y of a complex homogeneous space $Z = G/H$ influences the intersection of Y with other subvarieties of Z.

We consider a class of homogeneous spaces, rigged spaces, that includes Grassmannians, quadrics and $\mathbf{P}^r \backslash \mathbf{P}^k$ (the compliment in $\mathbf{P}^r$ of a linear subspace $\mathbf{P}^k$). A result of Corollary 4.5.2 is:

Let Z be a rigged homogeneous space with group G. Let Y be a compact smooth subvariety of Z possessing an ample normal bundle NY. (See [**10**] for the definition of ample.) Then the map

$$\phi_Y : \mathbf{P}(N^*Y) \to \mathbf{P}^a$$

determined by the G-sections of TZ is generically 1-1 (see 2.2 for the definition of ϕ_Y).

Corollary 4.5.2 and Theorem 5.2 imply that if X and Y are both smooth and compact subvarieties of Z with ample normal bundles, then for all $g \in G$, except for a closed codimension 2 subvariety of G, $X \cap g^{-1}(Y)$ is either a transverse intersection, or has precisely one singular point and it is non-degenerate quadratic.

In §5 these results are used to prove a generalized "second Lefschetz theorem on hyperplane sections", in analogy to the author's previous paper [**6**], and following the generalized first Lefschetz theorems of Barth [**2, 2A**] and Sommese [**19, 20**].

I expand, now, the outline of the paper.

Section 1 begins by considering a holomorphic bundle map $\psi: E \to F$ of holomorphic vector bundles over a complex space W, i.e. $\psi_x: E_x \to F_x$ is linear for all $x \in W$. The linear fibre space $\mathfrak{S}$ (see 4.1) is of central importance to the paper, and is defined as the kernel $\ker(g^*) := g^{*-1}$ (zero section of F) for a certain bundle map g^*. (The confusing notation "g^*" for the bundle map does not refer, of course, to any one element $g \in G$!) The map g^* fits into a commutative diagram of vector bundles (4.2.3) and the results of Lemma 1.4 allow us to conclude, by a vector