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NON-HAUSDORFF CONVERGENCE SPACES

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In this paper we will establish a method of removing the Hausdorff assumption from certain convergence space theorems. As specific applications the precise form of the closure of a compact set in a regular non-Hausdorff space is given and the exact relationship between cl and cl^2 in a non-Hausdorff compact regular space is obtained. Necessary and sufficient conditions that the transition space for this procedure be topological or pretopological are found and a few embedding theorems are obtained.

1. Introduction. Taking a hint from Thompson [**6**], who used the "Spiral" relation to investigate properties of topological maps with non-Hausdorff domains, let us define $(x, y) \in \mathrm{Sp}$ to mean there exists a filter which converges to both x and y. Now Sp is an equivalence relation if and only if the base space is *transitive* in the sense that if $\mathbf{F} \to x, y$ and $\mathbf{G} \to y, z$ then there exists a filter $\mathbf{H} \to x, z$. Since we want to take quotients by Sp our point of view is that only the class of transitive spaces will be considered. This class is quite broad as the next result shows.

PROPOSITION 1. *Regular spaces are transitive, spaces induced by uniform convergence structures are transitive, products of transitive spaces are transitive.*

Proof. If $\mathbf{F} \to x, y$ and $\mathbf{G} \to y, z$ then $\mathrm{cl}\,\mathbf{F} < \dot{y}$, $\mathrm{cl}\,\mathbf{G} < \dot{y}$ so $\mathrm{cl}\,\mathbf{F} \vee \mathrm{cl}\,\mathbf{G}$ exists and converges to x, z by regularity.

If the convergence is uniformizable [**1**] we may assume there is a base of symmetric filters each of which is coarser than the diagonal filter. Thus, if $\mathbf{F} \to x, y$ and $\mathbf{G} \to y, z$ there is some member Φ of the base such that each of the filters $\mathbf{F} \times \dot{x}$, $\mathbf{F} \times \dot{y}$, $\mathbf{G} \times \dot{z}$ is finer than Φ. So if $V \in \Phi$ then $V(x) \cap V(y) \neq \varnothing$ and $V(y) \cap V(z) \neq \varnothing$. A computation now shows $(x, z) \in V^4$ hence $\Phi^4(x) \to x$, and the space is transitive.

If filters converge, in the product, to f, g and g, h then use transitivity in each component $X(\lambda)$ to obtain filters which converge to $f(\lambda)$, $h(\lambda)$. Then the product filter converges to f, h. This ends the proof.