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ENGEL'S THEOREM FOR A CLASS OF ALGEBRAS

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A condition of nilpotency is derived for a class of algebras which include the almost alternative algebras of A. A. Albert. This result is seen to be an extension of Engel's theorem. Some consequences are then considered.

An algebra A over a Noetherian ring R is called almost alternative if it is power associative and satisfies the following identities

I.

$$u(vw) = \alpha_1(uv)w + \alpha_2 w(uv) + \alpha_3(vu)w + \alpha_4 w(vu) + \alpha_5(uw)v + \alpha_6 v(uw) + \alpha_7(wu)v + \alpha_8 v(wu)$$

and

II.

$$(vw)u = \beta_1 v(wu) + \beta_2(wu)v + \beta_3 v(uw) + \beta_4(uw)v + \beta_5 w(vu) + \beta_6(vu)w + \beta_7 w(uv) + \beta_8(uv)w$$

where u, v, $w \in A$ and α_i, $\beta_i \in R$. In what follows the requirement of power associativity will not be needed. Lie, alternative and (γ, δ)-algebras are contained in this class. It is the purpose of this note to give a criterion for nilpotency inspired by the Engel theorem in Lie algebras. However, it is not sufficient to assume that each multiplication is nilpotent. For let A be a 3-dimensional algebra generated by x, y and z where $xy = z$ and $zx = y$ and all other multiplications between basis elements are 0. Then A is a non-nilpotent algebra satisfying I and II with $\alpha_4 = \beta_4 = 1$ and all other α_i, $\beta_i = 0$. Also each right and left multiplication by any element in A is nilpotent. Note that A is not power associative.

Let R be a Noetherian ring. All algebras and modules over R are assumed to be unital. Let A be an algebra over R satisfying I and II. For each $x \in A$, R_x and L_x will denote right and left multiplication of A by x. Let M be an A-bimodule (see [**4**, p. 25]) with induced representation (S, T). Hence S and T satisfy identities derived from I and II and consequently S_{xy}, $T_{xy} \in \langle S_x, S_y, T_x, T_y \rangle$, the associative subalgebra of End(M) generated by S_x, S_y, T_x, T_y. In particular, if $x \in A$ and y is in the