PACIFIC JOURNAL OF MATHEMATICS
Vol. 107, No 1, 1983

VERSAL DETERMINANTAL DEFORMATIONS

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The paper investigates the deformations of a determinantal scheme arising from a deformation of the defining matrix. A sufficient condition is given for the parameter space of the versal deformation space to contain a unique smooth subscheme parameterizing determinantal deformations. Examples are given in which various determinantal representations of a scheme give different determinantal deformation spaces.

Versal deformation spaces for schemes of codimension 1 or 2 are well-understood, but the theory for schemes of codimension 3 is much more difficult, and most of the calculated examples fall into a category which we will call pre-image schemes, $X = f^{-1}(Y)$ for Y a rigid, Cohen-Macaulay scheme generated by minors of some generic matrix, with V and W ambiant affine spaces for X and Y and $\operatorname{codim}(X, V) = \operatorname{codim}(Y, W)$. The deformations of such schemes have been extensively studied for Y which is the generic determinantal, symmetric determinantal, or Pfaffian scheme [**1**, **2**, **3**, **6**]. Of these the most important are the determinantal schemes and to simplify our discussion we will restrict our attention to that case, although the methods would apply as well to the other types of schemes.

DEFINITION 1. If X is a determinantal scheme of type (a, b, c), with its ideal generated by the $c \times c$ minors of an $a \times b$ matrix M, then an M-deformation of X is one obtained by deforming the entries of M. More generally, for a preimage scheme $f^{-1}(Y)$, an f-deformation of X is one obtained by deforming the morphism f.

We will prove a sufficient condition for the versal M-deformation space to exist, and apply it to three of the important examples in which the versal deformation space is known: quotient singularities, monomial curves and coordinate axes.

EXAMPLE 1. The versal deformation space of the scheme of n coordinate axes in n-space has been calculated by D. S. Rim. The ideal is generated by $x_i \cdot x_j$, $i \neq j$, where $x_1, \ldots, x_n$ are the coordinates. This is, in