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EXPLICIT PL SELF-KNOTTINGS AND THE STRUCTURE OF PL HOMOTOPY COMPLEX PROJECTIVE SPACES

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We show that certain piecewise-linear homotopy complex projective spaces may be described as a union of smooth manifolds glued along their common boundaries. These boundaries are sphere bundles and the glueing homeomorphisms are piecewise-linear self-knottings on these bundles. Furthermore, we describe these self-knottings very explicitly and obtain information on the groups of concordance classes of such maps.

A piecewise linear homotopy complex projective space $\widetilde{CP}^n$ is a compact PL manifold M^{2n} homotopy equivalent to CP^n. In **[22]** Sullivan gave a complete enumeration of the set of PL isomorphism classes of these manifolds as a consequence of his Characteristic Variety theorem and his analysis of the homotopy type of G/PL. In **[15]** Madsen and Milgram have shown that these manifolds, the index 8 Milnor manifolds, and the differentiable generators of the oriented smooth bordism ring provide a complete generating set for the torsion-free part of the oriented PL bordism ring. Hence a study of the geometric structure of these exotic projective spaces $\widetilde{CP}^n$, especially with regard to their smooth singularities, may extend our understanding of the PL bordism ring. This paper begins such a study in which we obtain a geometric decomposition of $\widetilde{CP}^n$, into (for many cases) a union of smooth manifolds glued together by PL self-knottings on certain sphere bundles. We also obtain information on groups of concordance classes of PL self-knottings from these decompositions and a number of fairly explicitly constructed examples of self-knottings. I would like to thank by thesis advisor R. J. Milgram for many helpful discussions.

I. Sullivan's classification of PL homotopy $\widetilde{CP}^n$ proceeds as follows: Given a homotopy equivalence $h\colon \widetilde{CP}^n \to CP^n$ make h transverse regular to $CP^j \subset \widetilde{CP}^n$, the standard inclusion. The restriction of h to the transverse inverse image $h^{-1}(CP^j) = N^{2j} \subset \widetilde{CP}^n$ is a degree one normal map