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AFFINE CURVES OVER AN ALGEBRAICALLY NON-CLOSED FIELD

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In this paper, different $\bar{k}$-completions of a curve over an algebraically non-closed field k are compared. If the curve has k-points at infinity, then it is shown to admit a completion which is canonical. If $k = \mathbf{R}$ this is true also for rational curves.

Introduction. If $(V, \mathcal{O}_V)$ is an affine algebraic curve defined over a field k $(k \neq \bar{k})$ then V is the set of k-points of several non-isomorphic $\bar{k}$-curves which are called completions of V.

In this paper we compare these different completions and we prove (Theorem 3.1.) the existence of completions which, without creating new singularities, are extended as large as possible in the sense that they are not affine open sets of larger ones. These completions are called minimal. If a curve satisfies the condition of having k-points "at infinity" (cf. Theorem 3.5) then these minimal completions turn out to be all isomorphic. We shall say that "this is the canonical" completion of the curve. In the case $k = \mathbf{R}$ we prove that the above condition is also necessary if the genus of the curve is bigger than zero, while rational affine real curves always admit canonical complexification.

The involved techniques are mainly those of A. Tognoli (cf. [**6**]). We emphasize the use of affine representations (rather than that of completions (cf. §0)). First we introduce a suitable partial ordering in the set of isomorphism classes of affine representations; we are then able to show that the subset of affine representations which correspond to completions having no non-rational singularities is sufficiently rich and has minimal (up to isomorphism) elements.

0. We first recall some preliminaries. Throughout this paper k denotes a field and $\bar{k}$ an algebraic closure of it. Let V be an algebraic subset of k^n and let $\mathcal{O}_V$ denote the sheaf of regular functions defined over the open sets of V. It is known that

$$\Gamma_V = \Gamma(V, \mathcal{O}_V) = N_V^{-1}(k[X_1,\ldots,X_n]/\mathfrak{I}_V)$$

where $\mathfrak{I}_V = \{P \in k[X_1,\ldots,X_n] \mid P_{|V} \equiv 0\}$ and $N_V = \{g \in k[X_1,\ldots,X_n]/\mathfrak{I}_V \mid g(x) \neq 0 \text{ for each } x \in V\}$. More generally an affine