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SPLITTINGS OF FINITE GROUPS

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Let G be a group, written additively, M a set of integers, and S a subset of G. We will say that M and S form a splitting of G if every nonzero element of G has a unique representation as a product ms with $m \in M$ and $s \in S$, while 0 has no such representation. (Here "ms" denotes the sum of m s's if $m \geq 0$ and denotes $-((-m)s)$ if $m < 0$.) Splittings arise in connection with the problem of tiling Euclidean space by translates of certain unions of unit cubes, called "crosses" and "semicrosses".

In this paper, we develop a counting technique which gives information about S if M and G are known. This technique is used to reduce the study of splittings of finite abelian groups to those of nonsingular splittings and of purely singular splittings. (A splitting is nonsingular if every element of M is relatively prime to $|G|$; it is purely singular if, for every prime divisor p of $|G|$, some element of M is divisible by p.) Next, it is shown that every splitting of a noncyclic abelian p-group is nonsingular. A construction is then given which yields many purely singular splittings.

We then discuss a number of results and examples, including some infinite and nonabelian groups, and close with a list of open problems.

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