PACIFIC JOURNAL OF MATHEMATICS
Vol. 107, No. 1, 1983

EXTENSIONS OF d/dx THAT GENERATE UNIFORMLY BOUNDED SEMIGROUPS

RALPH J. DE LAUBENFELS

$A \equiv -d/dx$ on $BC_0(R^+)$, $\mathfrak{D}(A) \equiv \{f \in BC_0(R^+) \mid f' \in BC_0(R^+)\}$, is an example of a maximal accretive operator that does not generate a contraction semigroup. It does, however, have extensions that generate uniformly bounded semigroups. A large class of such extensions are presented. The same is done with d/dx and $-d/dx$ on $C_0[0, 1]$.

Introduction. The theory of accretive operators generalizes the theory of symmetric operators on a Hilbert space. An operator, T, on a Hilbert space is accretive if $\mathrm{Re}\langle Tx, x\rangle \geq 0$ for all x in the domain of T. A maximal accretive operator is one that has no proper accretive extensions. An m-accretive operator is one that generates a strongly-continuous contraction semigroup. The "m" suggests "maximal", because in a Hilbert space, every maximal accretive operator is m-accretive. (See Theorem 0.) This need not be true in a general Banach space. Lumer and Phillips, in 1961, ([**1**], p. 688) first gave an example of a maximal accretive operator that is not m-accretive. The space is $C_0[0, 1]$, and the accretive operator is d/dx, with domain $\{f \mid f' \text{ exists}, f' \in C_0[0, 1]\}$. Lumer and Phillips show that any proper extension of this operator fails to be accretive. To verify that this operator is not m-accretive, one uses the well-known ([**2**], p. 240) fact, that, if T is a closed accretive operator, then $(1 + T)$ is one-to-one, and T is m-accretive if and only if the range of $(1 + T)$ is the entire space. The range of $(1 + d/dx)$, with the domain given above, can be explicitly calculated to be $\{g \in C_0[0, 1] \mid \int_0^1 e^r g(r)\, dr = 0\}$, which is not dense in $C_0[0, 1]$. Intuitively, this operator fails to be m-accretive because d/dx should generate the translation semigroup $\{T_s\}_{s \geq 0}$, defined by $(T_s f)(t) = f(t - s)$, but this semigroup does not take $C_0[0, 1]$ into itself.

However, it is interesting that the Lumer-Phillips operator d/dx has extensions that generate uniformly bounded semigroups. This is the same as saying that there exist equivalent norms on $C_0[0, 1]$ with respect to which d/dx has m-accretive extensions. One of the main aims of this paper is to present such extensions.

The operator $-d/dx$, on $BC_0(R^+)$, with domain $\{f \mid f' \text{ exists}, f' \in BC_0(R^+)\}$, is a related example of a maximal accretive operator that is not m-accretive. Obvious modifications of the Lumer-Phillips proof ([**1**], p. 688) show that this operator has no proper accretive extensions, while the