

## ANNIHILATOR ALTERNATIVE ALGEBRAS

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**The aim of this paper is to use in the alternative case the axioms of the annihilator Banach algebras, and obtain some structure theorems similar to the known ones in the associative case.**

**1. Introduction.** As corollary of the main theorem of this paper: Theorem 3.11, we prove (see Corollary 3.12) that if  $A$  is a Kleinfeld semisimple annihilator complex purely-alternative algebra, then  $A$  is the closure of the direct sum of all its ideals which are isomorphic to the algebra of complex octonions  $\mathcal{O}_C$ .

**2. Prerequisites and notation.** In this paper,  $A$  is always understood to be an alternative algebra. "Ideal" without further qualification will mean "ideal of  $A$ ".

(2.1)  $N(A)$  denote the *nucleus* of  $A$  (for the definition of  $N(A)$ , see for example [3]).

We say an ideal (or right ideal or left ideal)  $I$  is nuclear provided  $\{0\} \neq I \subset N(A)$ .

(2.2) In what follows we write  $D_0$  for the *associator ideal* of  $A$ , and  $U_0$  its *maximun nuclear ideal*. It is known that  $U_0 D_0 = D_0 U_0 = \{0\}$  (see [6]).

(2.3) If  $I$  is an ideal, and  $I \cap D_0 = \{0\}$ , then  $I \subset U_0$ . A proof is easy.

(2.4) We say an alternative algebra is *purely-alternative* provided  $U_0 = \{0\}$  (See [6]).

(2.5) Say a right ideal  $I$  of  $A$  is trivial provided  $I \neq \{0\} = I^2$ , and say that  $A$  is *semiprime* provided  $A$  has no trivial right ideal.

$A$  is semiprime if and only if  $A$  contains no trivial ideal. (For a proof see [7]).

(2.6) For each right ideal  $R$ ,  $\text{Lan}(R)$  denotes the *left annihilator* of  $R$ :  $\text{Lan}(R) = \{a \in A: aR = \{0\}\}$ . If  $L$  is a left ideal,  $\text{Ran}(L) = \{a \in A: La = \{0\}\}$  is the *right annihilator* of  $L$ . If  $I$  is a right ideal of an ideal  $B$  of  $A$ , we note by  $\text{Lan}_B(I)$  the left annihilator of  $I$  in the algebra  $B$ .

(2.7) An element  $u$  of an algebra  $A$  is a *right modular unit* for a vector subspace  $E$  of  $A$  if  $\{a - au: a \in A\} \subset E$ .

A *modular left ideal* is a left ideal for which there exist a right modular unit.