PACIFIC JOURNAL OF MATHEMATICS
Vol. 107, No. 1, 1983

A GENERALIZATION OF THE GLEASON-KAHANE-ZELAZKO THEOREM

CHANG-PAO CHEN

In this paper, we consider two classes of commutative Banach algebras, which include $C^n(T)$, $\mathrm{Lip}_\alpha(T)$, $BV(T)$, $L^1 \cap L^p(G)$, $A^p(G)$, $L^1 \cap C_0(G)$, l^p, c_0, and $C_0(S)$. We characterize ideals of finite codimension in these two classes of algebras and thereby partially answer a question suggested by C. R. Warner and R. Whitley.

In [5] and [9], A. M. Gleason, J. P. Kahane and W. Zelazko gave independently the following characterization of maximal ideals: Let A be a commutative Banach algebra with unit element. Then a linear subspace M of codimension 1 in A is a maximal ideal in A if and only if it consists of noninvertible elements, or equivalently, each element of M belongs to some maximal ideal. This interesting result as first proved depended on the Hadamard Factorization Theorem.

This characterization of maximal ideals was extended in [15] and [16] to algebras without identity. In [16], C. R. Warner and R. Whitley also gave a characterization of ideals of finite codimension in $L^1(R)$ and $C[0, 1]$. They showed: Let A be any one of $L^1(R)$ and $C(S)$, where S is a compact subset of R. If M is a closed subspace of codimension n in A with the property that each element in M belongs to at least n regular maximal ideals, then M is an ideal. In fact, M is the intersection of n regular maximal ideals. Also in [16], C. R. Warner and R. Whitley suggested the following question: For what locally compact abelian group G does $L^1(G)$ have the property of $L^1(R)$ described above?

In this paper, we partially answer this question and generalize the work of C. R. Warner and R. Whitley. In this paper, two methods are introduced; One uses the Baire category theorem and the other generalizes the ideas of Theorems 2 and 4 in [16].

THEOREM 1. *Let A be a commutative Banach algebra with a countable maximal ideal space $\mathfrak{M}$. If M is a closed subspace of codimension n in A with the property that each element in M belongs to at least n regular maximal ideals, then M is an ideal, which is the intersection of n regular maximal ideals.*