PACIFIC JOURNAL OF MATHEMATICS
Vol. 107, No. 1, 1983

ON SUMS OF RUDIN-SHAPIRO COEFFICIENTS II

JOHN BRILLHART, PAUL ERDŐS AND PATRICK MORTON

Let $\{a(n)\}$ be the Rudin-Shapiro sequence, and let $s(n) = \sum_{k=0}^{n} a(k)$ and $t(n) = \sum_{k=0}^{n}(-1)^k a(k)$. In this paper we show that the sequences $\{s(n)/\sqrt{n}\}$ and $\{t(n)/\sqrt{n}\}$ do not have cumulative distribution functions, but do have logarithmic distribution functions (given by a specific Lebesgue integral) at each point of the respective intervals $[\sqrt{3/5}, \sqrt{6}]$ and $[0, \sqrt{3}]$. The functions $a(x)$ and $s(x)$ are also defined for real $x \geq 0$, and the function $[s(x) - a(x)]/\sqrt{x}$ is shown to have a Fourier expansion whose coefficients are related to the poles of the Dirichlet series $\sum_{n=1}^{\infty} a(n)/n^{\tau}$, where $\operatorname{Re}\tau > \frac{1}{2}$.

1. Introduction. In this paper we are concerned with the Rudin-Shapiro sums

$$s(x) = \sum_{k=0}^{[x]} a(k), \tag{1.1}$$

$$t(x) = \sum_{k=0}^{[x]} (-1)^k a(k), \tag{1.2}$$

where the numbers $a(k)$ are defined recursively by

$$a(2k) = a(k), \qquad a(2k+1) = (-1)^k a(k), \qquad k \geq 0, a(0) = 1. \tag{1.3}$$

An explicit formula for $a(k)$ is given by

$$a(k) = (-1)^{e(k)}, \tag{1.4}$$

where $e(k) = \sum_{i=0}^{s-1} \varepsilon_i \varepsilon_{i+1}$ and $k = \sum_{i=0}^{s} \varepsilon_i 2^i$, $\varepsilon_i = 0$ or 1. (See [**1**], Satz 1.)

The properties of these sums have been developed in [**1**], where it is shown that

$$\sqrt{\frac{3}{5}} < \frac{s(n)}{\sqrt{n}} < \sqrt{6}, \tag{1.5}$$

$$0 \leq \frac{t(n)}{\sqrt{n}} < \sqrt{3}, \tag{1.6}$$

for $n \geq 1$, and that the sequences $\{s(n)/\sqrt{n}\}$ and $\{t(n)/\sqrt{n}\}$ are dense in the intervals $[\sqrt{3/5}, \sqrt{6}]$ and $[0, \sqrt{3}]$.