PACIFIC JOURNAL OF MATHEMATICS
Vol 107, No 1, 1983

TRANSFORMATIONS OF CERTAIN SEQUENCES OF RANDOM VARIABLES BY GENERALIZED HAUSDORFF MATRICES

DAVID BORWEIN AND AMNON JAKIMOVSKI

Sufficient conditions are established for a generalized Hausdorff matrix to transform certain sequences of random variables into almost surely convergent sequences.

1. Introduction. Suppose that $\{X_n\}(n = 0, 1, \ldots)$ is a sequence of random variables defined on a probability space $(\Omega, \mathfrak{F}, P)$, and that $A = \{a_{nk}\}(n, k = 0, 1, \ldots)$ is an infinite matrix. Let

$$T_n = \sum_{k=0}^{\infty} a_{nk} X_k .$$

The following theorem concerning the almost sure convergence to zero of the sequence $\{T_n\}$ is due to Borwein [**1**].

THEOREM A. *If* $1 < p \leq 2$, $0 < M < \infty$ *and*
(1) $|X_n| \leq M$ *a.s. for* $n = 0, 1, \ldots,$
(2) $\sum_{0 \leq i_1 < i_2 < \cdots < i_n} |E(X_{i_1} X_{i_2} \cdots X_{i_n})|^{p/(p-1)} \leq M^n$ *for* $n = 1, 2, \ldots,$
(3) $\sum_{k=0}^{\infty} |a_{nk}| < \infty$ *for* $n = 0, 1, \ldots,$ *and*

$$\lim_{n \to \infty} \log n \left(\sum_{k=0}^{\infty} |a_{nk}|^p \right)^{1/(p-n)} = 0,$$

then $T_n \to 0$ *a.s.*

The sequence $\{X_n\}$ is said to be multiplicative if the expectation $E(X_{i_1} X_{i_2} \cdots X_{i_n}) = 0$ whenever $0 \leq i_1 < i_2 < \cdots < i_n$; in particular, it is multiplicative if it is independent with $EX_n = 0$ for $n = 0, 1, \ldots$. Condition (2) is trivially satisfied when $\{X_n\}$ is multiplicative. The nature of Theorem A is clarified by comparison with Kolmogorov's classical strong law of large numbers which states that if $\{X_n\}$ is independent with $EX_n = 0$ for $n = 0, 1, \ldots,$ and if

$$\sum_{k=0}^{\infty} \frac{EX_k^2}{(k+1)^2} < \infty, \quad \text{then } \frac{1}{n+1} \sum_{k=0}^{n} X_k \to 0 \quad \text{a.s.}$$

We shall denote by Γ_p the set of matrices A such that $T_n \to 0$ a.s. whenever the sequence $\{X_n\}$ satisfies conditions (1) and (2). Our primary