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KLEIN-GORDON SOLVABILITY AND THE GEOMETRY OF GEODESICS

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The Klein-Gordon equation is globally solvable on Lorentzian manifolds which have no imprisoned causal geodesics and which satisfy a certain convexity condition: for each compact subset K there exists a compact subset K' such that any causal geodesic segment with both endpoints in K lies in K'. This collection of Lorentzian manifolds includes many which are not globally hyperbolic. In any such manifold, the causal convex hull of a compact set is compact. When a curvature condition is satisfied, causally related points can be joined by at least one causal geodesic. A large class of these manifolds which fail to be globally hyperbolic may be constructed using warped products. The construction is independent of the warping function.

1. Introduction. In this paper we shall study Lorentzian geometries in which the inhomogeneous wave equation is globally solvable. Spacetimes which are globally hyperbolic comprise a class of examples of such geometries, but the class we shall study is much larger. Perhaps the simplest example which is not globally hyperbolic is a strip parallel to the time axis in the Minkowski plane.

Let (X, β) be a Lorentzian manifold. Here we take β to be a $(2,0)$-tensor, rather than the more usual $(0,2)$-tensor, for reasons which we now explain. The d'Alembertian operator is $\square := \operatorname{div}\operatorname{grad}\colon C^\infty(X) \to C^\infty(X)$. By polarization we may regard $\beta \in C^\infty(T^*X)$, so that the principal symbol of $\square$ may be identified as β. Recall that T^*X is a symplectic manifold in a natural way. For the discussion of global solvability, certain integral curves of the Hamiltorian vector field H^β play the decisive role. These are the *bicharacteristic strips*, those along which $\beta = 0$. Their projections in X are the *bicharacteristics* and are, up to parametrization, the null geodesics of β. When β is not smooth, this is vital **[11]**.

To have any hope of solving the inhomogeneous wave equation

$$\square u = f$$

we must allow u to be distributional (in the sense of Schwartz), and then we might as well let f be distributional too. In general, there are distributions $\mathfrak{D}'(X)$ and twisted distributions $\mathfrak{D}'_1(X)$. We shall use the natural