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THE DUAL OF THE NILRADICAL OF THE PARABOLIC SUBGROUPS OF SYMPLECTIC GROUPS

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For an arbitrary parabolic subgroup P of the real or complex symplectic group, let N be the nilradical. Using Kirillov theory, a subset of the dual of N is found, whose complement has Plancherel measure zero. It is shown how these representations extend by combining with the oscillator representation of a lower rank symplectic group. A result is obtained concerning the commuting algebra of the restrictions to P of the principal series representation of the symplectic group induced from a unitary character of the opposite parabolic.

Introduction. In [**6**] and [**2**] there are irreducibility theorems for principal series representations of symplectic groups induced from unitary characters of certain maximal parabolic subgroups. Such a representation can be realized to act in the L^2-space of a nilpotent subgroup, the nilradical of the opposite parabolic. The irreducibility results are obtained in two stages. In the first stage the representation T is restricted to the opposite parabolic and the commuting algebra of the restriction is computed using nilpotent harmonic analysis. Since these parabolics are maximal subgroups, the full symplectic group is generated by the opposite parabolic together with a single element, say p. The commuting algebra of T is, therefore, the subalgebra of the commuting algebra of the restriction consisting of operators that commute with $T(p)$. The second stage is the difficult determination of which operators these are. For arbitrary parabolic subgroups of the symplectic groups, even for arbitrary maximal ones, it appears that the second stage of this program is not feasible and the irreducibility theorems must come from more powerful methods in semisimple representation theory. However, the first stage can be carried out in full generality, and it is of interest for the way in which the oscillator representation occurs and because of the computations involved in the nilpotent harmonic analysis. This is the topic of this paper.

To be more specific, let P be a parabolic subgroup of the symplectic group $\mathrm{Sp}(n, F)$, where $F = \mathbf{R}$ or $\mathbf{C}$. It is known that the principal series representation T of $\mathrm{Sp}(n, F)$ induced up from a unitary character of P can be realized to act in $L^2(N)$, where N is a nilpotent subgroup of $\mathrm{Sp}(n, F)$ and $N \cap P$ is trivial. Let M be the normalizer of N in P, then the semidirect product NM is a parabolic subgroup conjugate to P (NM is