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ABSOLUTELY FLAT SEMIGROUPS

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All left modules over a ring are flat if and only if the ring is von Neumann regular. In [7], M. Kilp showed that for a monoid S to be left absolutely flat (i.e., for all left S-sets to be flat) regularity is necessary but not sufficient. Kilp also proved [8] that every inverse union of groups is absolutely flat. In the present paper we show that in fact every inverse semigroup is absolutely flat and that the converse is not true.

1. Preliminaries. We consider a monoid to be a universal algebra $(S;\ \cdot, 1)$ of type $(2,0)$. We shall consistently denote such a monoid by S and on occasion consider it to be a semigroup via the forgetful functor. If S is a monoid S-*Ens* (respectively, *Ens*-S) will denote the class of left (right) unital S-sets. In §§1 and 2, we deal only with monoids and their associated S-sets. In §3 the considerations will be extended to arbitrary semigroups.

Let S be a monoid. For $A \in$ *Ens*-S and $B \in S$-*Ens*, let τ denote the smallest equivalence relation on $A \times B$ containing all pairs $((as, b), (a, sb))$ for $a \in A$, $b \in B$, and $s \in S$. The tensor product $A \otimes B$ (or, more precisely, $A \otimes_S B$) is defined to be the set $(A \times B)/\tau$, and possesses the customary universal mapping property with respect to balanced maps from $A \times B$ to an arbitrary set. For $a \in A$ and $b \in B$, $a \otimes b$ represents the τ-class of (a, b).

The following information will be useful in the sequel. If S is any monoid and $s, t \in S$ then $\theta(s, t)$ will denote the principal left congruence on S identifying s and t. It is easy to check that for u, v in S, $(u, v) \in \theta(s, t)$ if and only if either

$$u = v$$

or

there exist $w_1, \ldots, w_n, s_1, \ldots, s_n, t_1, \ldots, t_n \in S$ where $\{s_i, t_i\} = \{s, t\}$ for $i = 1, \ldots, n$, such that

$$u = w_1 s_1,$$
$$w_1 t_1 = w_2 s_2,$$
$$\vdots$$
$$w_n t_n = v.$$