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CHARACTERIZATIONS OF COMPLETELY NONDETERMINISTIC STOCHASTIC PROCESSES

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A discrete weakly stationary Gaussian stochastic process $\{x(t)\}$, is completely nondeterministic if no non-trivial set from the σ-algebra generated by $\{x(t): t > 0\}$ lies in the σ-algebra generated by $\{x(t): t \leq 0\}$. In [8] Levinson and McKean essentially showed that a necessary and sufficient condition for complete nondeterminism is that the spectrum of the process is given by $|h|^2$ where h is an outer function in the Hardy space, H^2, of the unit circle in C with the property that $h/\bar{h}$ uniquely determines the outer function h up to an arbitrary constant. In this paper we consider several characterizations of complete nondeterminism in terms of the geometry of the unit ball of the Hardy space H^1 and in terms of Hankel operators.

1. Introduction. In **[10]** Sarason defines a property of a discrete weakly stationary Gaussian stochastic process, $\{x(t)\}$, which he calls complete nondeterminism. This condition is that no set from the future of the process (i.e. the σ-algebra generated by the random variables $x(t)$ for $t > 0$) lies in the past (i.e. the σ-algebra generated by $x(t)$ for $t \leq 0$), except for null sets and the complements of null sets. In the spectral representation this condition becomes the following. Let m be the spectral measure of the process and let $\mathcal{P}$ denote the span in $L^2(m)$ of the exponentials $e^{in\theta}$ with $n \leq 0$ where functions are defined on $\mathbf{T}$, the unit circle in $\mathbf{C}$. Let $\mathcal{F}$ denote the span in $L^2(m)$ of the exponentials $e^{in\theta}$ with $n > 0$. Then complete nondeterminism is equivalent to the condition that $\mathcal{P} \cap \mathcal{F} = \{0\}$. It is clear that this condition reflects a certain kind of independence (in a statistical sense) of the past, $\mathcal{P}$, and the future, $\mathcal{F}$.

It is of interest to characterize those measures m on $\mathbf{T}$ which lead to completely nondeterministic (cnd) processes. In **[10]** a necessary and sufficient condition for complete nondeterminism was stated as the measure m being absolutely continuous with respect to Lebesgue measure, $d\theta$, with $\log(dm/d\theta)$ integrable. Unfortunately this characterization is incorrect. In [**8**, p. 105] Levinson and McKean essentially describe a partial characterization of cnd processes which we discuss in §3. This paper