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BESSEL FUNCTIONS ON P_n

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In this paper Bessel functions are defined in the homogeneous symmetric space $GL(n,\mathbf{R})/O(n)$. Two definitions are given. One is an integral over the space itself, the other is a Euclidean integral. A relation between the integrals is proved. The use of this relation is shown explicitly in a low dimensional case. Some additional properties of these Bessel functions are then noted.

I. Introduction. The Riemannian symmetric space P_n of $n \times n$ symmetric matrices over the real numbers has appeared in many areas of mathematics. It has been studied in connection with multivariate statistics [**J**], [**I**], with analytic number theory [**S**], [**M**], and with the study of other higher rank symmetric spaces, such as the Siegel upper half space of $n \times n$ complex matrices with imaginary part in P_n. The above references have been cited merely as examples of the uses of P_n.

One way to approach questions about P_n is to use harmonic analysis and special functions. P_n has many coordinate systems. In the geodesic polar coordinate system a general theory has been worked out by Harish-Chandra [**HC**] and Helgason [**Hel**]. Using some computations of Bhanu-Murti [**BM**], Audrey Terras has been able to make this theory explicit for P_n [**T**]. However, it seems that the so-called partial Iwasawa coordinate system is needed in the study of some questions that arise in number theory. This is the coordinate system that is used in this note. In fact, the functions and formulas proved here have already found application in the Fourier series expansion of Eisenstein series for $GL(3,\mathbf{Z})$ due to recent work of Kaori Imai and Audrey Terras [**I-T**].

The purpose of this note will be to define some Bessel functions for P_n and prove some of their properties. To begin, however, we establish some notation and recall some basic facts about P_n. We will only mention those details of structure that directly concern us here.

II. Basic facts and notation. Let P_n be the space of all $n \times n$ symmetric matrices over the real numbers. Let $Y = (y_{ij})$ be in P_n and let A be in $GL(n,\mathbf{R})$. Then $GL(n,\mathbf{R})$ acts on P_n by sending Y to $Y[A] = {}'AYA$ where ${}'A$ denotes the transpose of A. The orthogonal matrices $O(n)$ fix the identity I in P_n. The action is sufficiently nice that P_n can be identified as the symmetric space $K\backslash G$ where $G = GL(n,\mathbf{R})$ and $K = O(n)$. The