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REDUCTION OF ELLIPTIC CURVES OVER IMAGINARY QUADRATIC NUMBER FIELDS

R. J. STROEKER

It is shown that an elliptic curve defined over a complex quadratic field K, having good reduction at all primes, does not have a global minimal (Weierstrass) model. As a consequence of a theorem of Setzer it then follows that there are no elliptic curves over K having good reduction everywhere in case the class number of K is prime to 6.

1. Introduction. An elliptic curve over a field K is defined to be a non-singular projective algebraic curve of genus 1, furnished with a point defined over K. Any such curve may be given by an equation in the Weierstrass normal form:

$$y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6 \tag{1.1}$$

with coefficients a_i in K. In the projective plane $\mathbf{P}_K^2$, the point defined over K becomes the unique point at infinity, denoted by $\underline{0}$. Given such a Weierstrass equation for an elliptic curve E, we define, following Néron and Tate ([**12**], §1; [**6**], Appendix 1, p. 299):

$$\begin{cases} b_2 = a_1^2 + 4a_2, & c_4 = b_2^2 - 24b_4, \\ b_4 = a_1a_3 + 2a_4, & c_6 = -b_2^3 + 36b_2b_4 - 216b_6, \\ b_6 = a_3^2 + 4a_6, & \\ b_8 = a_1^2a_6 - a_1a_3a_4 + 4a_2a_6 + a_2a_3^2 - a_4^2, & \\ \Delta = -b_2^2b_8 - 8b_4^3 - 27b_6^2 + 9b_2b_4b_6, & j = c_4^3/\Delta. \end{cases} \tag{1.2}$$

The discriminant Δ, defined above, is non-zero if and only if the curve E is non-singular. In particular, we have

$$4b_8 = b_2b_6 - b_4^2 \quad \text{and} \quad c_4^3 - c_6^2 = 2^6 3^3 \Delta. \tag{1.3}$$

The various representations of an elliptic curve over K, with the same point at infinity, are related by transformations of the type

$$\begin{matrix} x = u^2x' + r \\ y = u^3y' + u^2sx' + t \end{matrix} \quad \text{with } r, s, t \in K \text{ and } u \in K^*. \tag{1.4}$$