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CHARACTERISTIC CLASSES FOR SPHERICAL FIBRATIONS WITH FIBRE-PRESERVING FREE GROUP ACTIONS

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Let $G(H)$ be the monoid of H equivariant self maps of $S(nV)$, the unit sphere of n copies of a finite dimension orthogonal representation V of a finite group H, stabilized over n in an appropriate way. Let $SG(H)$ be the submonid of $G(H)$ consisting of all degree 1 maps. If H_1 is a subgroup of H there is a natural forgetful map $SG(H) \to SG(H_1)$ and if Z is the center of H there is a natural action map $BZ \times SG(H) \to SG(H)$ induced by the natural action of Z on H. The main results of this paper are the calculations of the Hopf algebra structures of $H_*(SG(\mathbf{Z}/p^n), \mathbf{Z}/p)$ and $H_*(BSG(\mathbf{Z}/p^n), \mathbf{Z}/p)$ for all n and all primes p, the calculations in homology of forgetful maps induced by the natural inclusions $\mathbf{Z}/p^{n-1} \to \mathbf{Z}/p^n$ and, for $H = \mathbf{Z}/2$, the calculation of the action map $H_*(\mathbf{R}P^\infty, \mathbf{Z}/2) \otimes H_*(BSG(\mathbf{Z}/2), \mathbf{Z}/2) \to H_*(BSG(\mathbf{Z}/2), \mathbf{Z}/2)$.

Introduction. In this paper we study oriented spherical fibrations modeled on an orthogonal representation V of a finite group H, a theory first studied by Segal [**25**], and Becker and Schultz [**2**]. More precisely we consider fibrations with a fibre-preserving action of H on the total space such that the fibre is H equivariantly homotopy equivalent to the unit sphere $S(V)$ of V. We stabilize these fibrations by forming the fibrewise join with the trivial $S(V)$ bundle. As we have specific geometric applications in mind, we concentrate on the case where $S(V)$ is *H-free*.

Well known results of Barrett, Gugenheim, and Moore [**1**], May [**15**], Stasheff [**29**], and Waner [**35**], reduce this question to the study of the homotopy-type of the classifying space of the submonoid $SG(H)$ of degree 1 maps in the space

$$G(H) = \varinjlim_{n} \operatorname{Map}_H(S(nV), S(nV))$$

of stable equivariant self-maps of $S(nV)$. This is the model studied by Becker and Schultz [**2**]. They showed for any compact Lie group H, that $G(H)$ is homotopy equivalent to $Q(BH^\zeta)$, where $Q(X)$ is the union over n of $\Omega^n\Sigma^n X$, and BH^ζ is the Thom space of the vector bundle ζ over BH associated to the adjoint representation of H on its Lie algebra. Notice that if H is finite then $\zeta = 0$, and $BH^\zeta = BH^+$, the disjoint union of BH