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FACTORIZATION THEOREMS FOR DIFFERENT CLASSES OF ANALYTIC FUNCTIONS IN MULTIPLY CONNECTED DOMAINS

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This paper consists of four sections. In the first section we give a survey on the reproducing kernel for harmonic functions in finitely-connected Jordan regions. We also prove a certain version of Fatou's theorem which we will use in the next sections.

In the second part we construct the generalized Schwarz kernel for an arbitrary finitely-connected Jordan domain. This kernel reproduces any continuous single-valued analytic function inside the domain by the boundary values of its real part. Also, we give an explicit formula for the real part of this kernel in terms of the harmonic measures.

In the third section we study the Blaschke products in arbitrary Jordan domains.

The main results are contained in the fourth section. There we prove factorization theorems for the classes N, N_+, H_p and E_p.

Introduction. It is well known that R. Nevanlinna's and V. I. Smirnov's factorization theorems have been very useful for many problems concerning analytic functions in the unit disc (e.g. see [**8**], [**9**], [**15**], [**19**]). Unfortunately, a direct attempt to extend these results to multiply connected domains has been unsuccessful. The fact is that the most natural function to play the role of the Schwarz kernel in such domains is not single-valued. Another problem appearing in that case is to define the Blaschke factor. It is obvious that even in an annulus one cannot find a single-valued function $f(z)$ satisfying the following properties: (1) $f(z)$ vanishes only at one given point; (2) $f(z)$ is continuous up to the boundary; (3) $|f(z)|$ is equal to 1 on the whole boundary of the annulus.

Many papers investigating the classes of analytic functions in multiply-connected domains have appeared. We refer the reader to the survey by S. Ya. Havinson and G. C. Tumarkin [**10**] which is quite detailed. The construction of the "Blaschke products" in finitely connected regions was suggested by V. A. Zmorovič. The convergence theorem for the products of that type has been proved by P. M. Tamrazov in [**22**].

The first attempts to generalize the Schwarz formula to finitely connected domains had already taken place in the 19th century. (By the