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ON F-SPACES AND F'-SPACES

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Two problems concerning F-spaces and F'-spaces are investigated. The first problem is to characterize those F-spaces whose product with every P-space is an F-space. A new necessary condition is obtained which is in fact a characterization of those F-spaces whose product with any P-space with only one non-isolated point is an F-space. As a corollary an example of a locally compact F-space and a P-space whose product is not an F-space is obtained. The second problem is to verify a conjecture of Comfort, Hindman and Negrepontis. It is shown that each weakly Lindelöf F'-space is an F-space. Also, each zero-dimensional weakly Lindelöf F'-space is strongly zero-dimensional.

0. Introduction. All spaces considered are completely regular and βX is the Čech-Stone compactification of X. A space X is: an *F-space* if disjoint cozero subsets of X are contained in disjoint zero sets; a *P-space* if each zero set is closed and open; an *F'-space* if disjoint cozero subsets have disjoint closures; and *basically disconnected* (BD) if each cozero set has clopen closure. In 1960, Curtis [**Cu**] showed that if a product of spaces is an F-space then it can be expressed as a product of a P-space with an F-space. The interesting question is then to characterize those F-spaces Y so that $X \times Y$ is an F-space for each P-space X. In [**CHN**], the analogous question for F'-spaces and BD spaces is solved. In [**N**], Negrepontis showed that Y be compact is sufficient and Hindman [**H**] weakened this considerably. A space Y is *weakly Lindelöf* if each open cover of Y has a countable subcollection whose union i sdense. Hindman's condition is a weakening of weakly Lindelöf.

The only necessary condition on an F-space Y to arise is from the characterization for F'-spaces. Clearly if a space is an F-space then it must also be an F'-space. In [**H**] it is shown that this condition is not sufficient. We obtain a new necessary condition on Y in order that $X \times Y$ is an F-space for each P-space X. This condition is actually a characterization of those F-spaces whose product with each P-space with a unique non-isolated point is an F-space. As a consequence of this we are able to construct an example of a locally compact F-space Y and a P-space X such that $X \times Y$ is not an F-space. In addition this provides an example of a locally compact F'-space which is not an F-space, answering a question related to the author by M. Henriksen.