

THE DETERMINANTAL IDEALS OF LINK MODULES. II

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Let H be the multiplicative free abelian group of rank $m \geq 1$. Suppose $0 \rightarrow B \rightarrow A \rightarrow IH \rightarrow 0$ is a short exact sequence of $\mathbf{Z}H$ -modules, and the module A is finitely generated. Then B is also a finitely generated $\mathbf{Z}H$ -module, and for any $k \in \mathbf{Z}$ the determinantal ideals of A and B satisfy the equality

$$E_k(A) : (IH)^p = E_{k-1}(B) : (IH)^q$$

for all sufficiently large values of p and q . Furthermore, if this exact sequence is the link module sequence of a tame link of m components in S^3 , then

$$E_k(A) = E_{k-1}(B) : (IH)^{\binom{m-1}{2}}$$

whenever $k \geq m$.

1. Introduction. Let H be the multiplicative free abelian group of rank $m \geq 1$, and $\mathbf{Z}H$ its integral group ring; if $\epsilon: \mathbf{Z}H \rightarrow \mathbf{Z}$ is the augmentation map then its kernel is the augmentation ideal IH of $\mathbf{Z}H$. Following [6], we will call a short exact sequence

$$(1) \quad 0 \rightarrow B \xrightarrow{\phi} A \xrightarrow{\psi} IH \rightarrow 0$$

of $\mathbf{Z}H$ -modules and homomorphisms an *augmentation sequence*, provided that the $\mathbf{Z}H$ -module A is finitely generated. The module B is then also finitely generated, and so for any $k \in \mathbf{Z}$ there are well-defined determinantal ideals $E_k(A)$, $E_k(B) \subseteq \mathbf{Z}H$.

In [6] we discussed the relationship between the product ideals $E_k(A) \cdot (IH)^p$ and $E_{k-1}(B) \cdot (IH)^q$ for various values of k , p , and q . In the present paper, instead, we will consider the relationship between the various quotient ideals $E_k(A) : (IH)^p$ and $E_{k-1}(B) : (IH)^q$. (We recall the definition: if $U, V \subseteq \mathbf{Z}H$ are ideals then the quotient ideal $U : V$ is $\{x \in \mathbf{Z}H \mid xV \subseteq U\}$.)

At first glance, it may seem that if $U \subseteq \mathbf{Z}H$ is an ideal the quotient ideals $U : (IH)^p$ and the various product ideals $U \cdot (IH)^q$ are, in some