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HARMONIC ANALYSIS OF SPHERICAL FUNCTIONS ON REDUCTIVE GROUPS OVER $\mathfrak{p}$-ADIC FIELDS

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We introduce spaces $\mathcal{C}^{\gamma}(G, K)$, $0 < \gamma \leq 2$, of rapidly decreasing K-bi-invariant functions on a reductive $\mathfrak{p}$-adic group G where K is a maximal compact subgroup of G, and we study spherical transformations on these spaces. The image of $\mathcal{C}^{\gamma}(G, K)$ under spherical transformation is completely described when K is a standard maximal compact subgroup of G.

Introduction. Let G be the group of rational points of a connected reductive algebraic group defined over a locally compact totally disconnected nondiscrete field k, with anisotropic center.

The group G is a totally disconnected locally compact group. A maximal compact subgroup K of G is fixed. The set of all (zonal) spherical functions on G with respect to K is parametrized naturally by orbits of the Weyl group W in a commutative complex Lie group $\hat{T}_{\mathbf{C}}$.

For $0 < \gamma \leq 2$ a family of Schwartz spaces $\mathcal{C}^{\gamma}(G, K)$ of rapidly decreasing K-bi-invariant functions on G is defined. It is shown that they are Fréchet algebras under convolution. For $f \in \mathcal{C}^{\gamma}(G, K)$ its spherical transform is defined by

$$\hat{f}(s) = \int_G f(g)\Gamma_s(g^{-1})\,dg$$

for $s \in \hat{T}$, where $\hat{T}$ is the greatest compact subgroup of $\hat{T}_{\mathbf{C}}$ and Γ_s is the spherical function corresponding to s. $\hat{T}$ is a compact real Lie group.

Let $C_W^{\infty}(\hat{T})$ denote the algebra of all W-invariant infinitely differentiable functions on $\hat{T}$ under pointwise multiplication. Then the spherical transformation $f \mapsto \hat{f}$ is a continuous epimorphism of $\mathcal{C}^2(G, K)$ onto $C_W^{\infty}(\hat{T})$. With certain additional conditions on K, we show that the spherical transformation is an isomorphism of topological algebras. The situation when these additional conditions are fulfilled is called the standard case and the contrary case is called the exceptional [**5**]. A standard maximal compact subgroup of G may always be found. If G is a split group, these additional conditions are always fulfilled.

If $\gamma < 2$, every spherical transform $\hat{f}$ of $f \in \mathcal{C}^{\gamma}(G, K)$ may be uniquely extended to a holomorphic function defined on a W-invariant domain D^{γ}