COHOMOLOGY OF COMODULES

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The purpose of the present paper is to develop some aspects of a cohomology theory in the category of C comodules, where C is a bialgebra. This cohomology theory generalizes the rational cohomology of affine algebraic groups and also the cohomology of Lie Algebras. We develop the theory up to the point where we can guarantee the convergence of the analogue of the Hochschild Serre spectral sequence and the convergence of another spectral sequence, which is a natural generalization of the result known as Shapiro's lemma in the cohomology theory of finite groups.

1. Introduction. The attempts to generalize the Hochschild-Serre (or Lyndon-Hochschild-Serre) spectral sequence for the rational cohomology have a long history. One of the main difficulties was the fact that it is no longer evident (as it is for finite groups) that if K is a closed normal subgroup of G then the algebra $P(G)$ of polynomial functions on G is injective as a K-module. A first remark about the need to prove that result was made by Hochschild in [4]. Recently, Cline, Parshall and Scott in [1] and Habousch in [3] proved the above injectivity result and established the validity of the spectral sequence. Here we prove a generalization of the above result, namely that if C and D are commutative Hopf Algebras over a field and $\pi\colon C \to D$ is a normal surjective Hopf Algebra map, then C is injective as a D-comodule (Theorem 4.13).

We begin with a brief description of each section:

2. Cohomology of comodules. Here we introduce the cohomology theory with which we shall be concerned throughout the paper. We present it axiomatically and also construct an explicit resolution functor whose homology gives us the cohomology of comodules. The definition of this comodule cohomology seems to have been known to some specialists and was communicated to the author by G. Hochschild.

3. Normal maps and actions on the cohomology. This section is of a technical nature and has the main purpose of establishing the generalization to the category of comodules of the conjugation action of a group G on the cohomology $H^i(K, M)$ where M is a G-module and K a normal subgroup of G.