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REALIZING CENTRAL DIVISION ALGEBRAS

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Let D be a finite dimensional division algebra over the rational field. We consider the question: for which primes p is D isomorphic to the quasi-endomorphism algebra of a p-local torsion free abelian group G whose rank is equal to the dimension of D? We show that D can be realized in this way for exactly those primes p such that $\hat{Q}_p \otimes D$ is not a product of division algebras.

1. Introduction. The question "which finite dimensional algebras over the field of rationals Q can be realized as quasi-endomorphism algebras of finite rank torsion free groups?" was first posed in [**3**]. The answer "all such algebras" came two years later in [**6**] as a corollary to Corner's Theorem: If R is a reduced, torsion free ring with rank $R = n < \infty$, then R is isomorphic to the endomorphism ring of a torsion free group G of rank $2n$. Corner also showed that it is not always possible to realize such a ring by the endomorphisms of a group of rank less than $2n$. However, in [**12**] Zassenhaus showed that if R is free as an abelian group, then the group G could be chosen to have rank n. Butler [**5**] showed that the same result is true under the weaker hypothesis that R is locally free. It follows from the theorems of Zassenhaus and Butler that every n dimensional rational algebra is the quasi-endomorphism algebra of a group G of rank n. This paper considers the question of what occurs when G is required to be p-local, that is $qG = G$ for all primes $q \neq p$.

Problem. For a finite dimensional, rational divison algebra D find all primes p such that there is a p-local group G with rank G = dimension D and with D isomorphic to the ring of quasi-endomorphisms of G.

Our main result is that such a group G exists for exactly those primes p such that $\hat{Q}_p \otimes D$ is not a product of division algebras.

§§2, 3, and 4 of the paper set up some machinery that is used to construct groups with the required properties. The ideas described in these Sections are variations on standard themes, but for convenience, the proofs of the needed results are sketched. The main theorem is proved in §5.

NOTATION. The symbols Z, Q, F_p, $\hat{Z}_p$, and $\hat{Q}_p$ respectively denote the ring of integers, the field of rational numbers, the prime field of order p,