PACIFIC JOURNAL OF MATHEMATICS
Vol 109, No 1, 1983

WEAK APPROXIMATION OF STRATEGIES IN MEASURABLE GAMBLING

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In a measurable gambling house, the measurable strategies available at a fortune f are weakly dense in the collection of all strategies available at f.

1. Background. In §2.16 of their monograph on the theory of gambling, Dubins and Savage [**3**] showed how the imposition of regularity conditions on a gambling problem yields regularity properties about its solution. In particular, they established certain continuity conditions which guarantee the existence of Borel-measurable, nearly-optimal strategies. Strauch [**9**] defined the notion of measurable gambling house and showed that if a measurable house is leavable, then the optimal return function is universally measurable and good measurable strategies exist. Without assuming leavability, Sudderth [**10**] investigated measurable gambling problems and showed, among other things, that if the gambler's payoff, or utility, under a strategy $\hat{\sigma}$ is $\int g\,d\sigma$, where g is a bounded, finitary function, then the gambler may, without loss, restrict himself to measurable strategies.

However, the question of adequacy of measurable strategies for measurable problems — with a strategic utility function of the type introduced by Dubins and Savage — remained unanswered. (See remarks in [**5**].) Recently, though, Purves and Sudderth [**8**] established certain approximation results which provide a greater understanding of the optimal return function and, at least in some cases, imply its universal measurability.

In this note, we clarify the position of the set of measurable strategies available at a fortune f within the set of all strategies available at f by showing that the former set is "weakly dense" in the latter. This density property, however, is not strong enough to answer the question of adequacy of measurable strategies.

We avoid entirely any discussion of optimality and, instead, examine the strategic measures directly. We demonstrate that if $A_1, A_2, \ldots, A_n$ are finitary, Borel subsets of the space of histories, and if σ is an available strategy, then there is an available measurable strategy $\hat{\sigma}$ which makes