

## SECTIONAL REPRESENTATIONS OF BANACH MODULES

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**This paper is concerned with the representation of Banach algebras and Banach modules as sections of bundles of Banach spaces. In particular: (1) if  $A$  is a commutative Banach algebra, then  $A$  may be represented by sections of a locally trivial canonical line bundle; (2) if  $A$  is a Banach algebra which is represented as sections of a canonical bundle of Banach algebras, then there is a natural way in which any Banach module  $M$  over  $A$  can be represented by sections of a canonical bundle of Banach modules over the corresponding algebra bundle. We also investigate projective tensor products of bundles of Banach algebras and bundles of Banach modules.**

The present paper extends and generalizes earlier results to be found in [7] and [8], to which the reader is referred for terminology and notations. Both of these papers center upon the study of sectional representations of “Gelfand type” for Banach modules  $(M, A)$  such that

- (1) the algebra  $A$  is commutative, and
- (2) the module satisfies the (KR) condition.

Such modules must be essential, and conversely, every essential module over an algebra with bounded approximate identities satisfies the (KR) condition. In the present paper it is shown that the (KR) condition can be dropped altogether provided one is willing to accept a somewhat altered version of the classical Gelfand representation of  $A$ . Rather than represent  $A$  by complex-valued functions, one can represent  $A$  by sections of a canonical line bundle. This “primitive” Gelfand representation is described in §1, which concerns bundles of Banach algebras more generally.

§2 concerns sectional representation of Banach modules. A new construction for canonical bundles of Banach modules is given (Theorem 2.3), which generalizes various known constructions. Basically, Theorem 2.3 states that whenever one has a representation of a (possibly noncommutative) Banach algebra  $A$  by sections of a canonical algebra bundle, then there is a natural means whereby one can represent every module  $M$  over  $A$ , namely,  $M$  can be represented by sections of a canonical bundle of Banach modules over the corresponding algebra bundle. This result applies, in particular, to the primitive Gelfand representation of a commutative algebra. It applies also to the sectional representations of  $C^*$ -algebras which have been studied by Hofmann, Varela, Dupré, et al. Moreover, the