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SOME GENERALIZATIONS OF CONTRACTION THEOREMS FOR FOURIER SERIES

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This paper deals with some contraction theorems for Fourier series and certain properties of the Fourier coefficients of functions in the Lipschitz spaces.

1. We shall assume that functions f and g are integrable in $(-\pi, \pi)$ and periodic with period 2π. Denote their Fourier series by

$$f \sim \sum_{n=-\infty}^{\infty} c_n e^{inx}, \quad \text{and} \quad g \sim \sum_{n=-\infty}^{\infty} d_n e^{inx}.$$

According to B. S. Yadav **[10]**, we say a function g is a shrivel contraction of order j of a function f if it holds that

$$(1) \qquad |L^{(j)}(t, x, g)| \leq K |L^{(j)}(t, x, f)|,$$

where

$$L^{(j)}(t, x, f) = \frac{1}{t}\int_0^t \Delta_u^j f(x)\, du$$

and

$$\Delta_u^j f(x) = \sum_{m=0}^{j} (-1)^{j+m} \binom{j}{m} f(x + (j-2m)u),$$

which is the symmetric difference of order j of $f(x)$ with respect to u.

Yadav proved the following:

THEOREM Y. *If g is a shrivel contraction of order j of f and if*

$$(2) \qquad {}_2B_p(c_n) = \left[\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} |n|^{-p/2} \left(\sum_{|k|\geq n+1} |c_k|^2\right)^{p/2}\right]^{1/p} < \infty,$$

then

$$\|d_n\|_p^p = \sum_{n=-\infty}^{\infty} |d_n|^p < \infty, \quad \textit{where } 0 < p \leq 2.$$