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THE WHITTAKER MODELS OF INDUCED REPRESENTATIONS

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If F is a local non-Archimedean field, then every irreducible admissible representation π of $\mathrm{GL}(r, F)$ is a quotient of a representation ξ induced by tempered ones. We show that ξ has a Whittaker model, even though it may fail to be irreducible.

1. Introduction and notations.

(1.1) Let F be a local non-Archimedean field and ψ an additive character of F. Let G be the group $\mathrm{GL}(2, F)$ and B the subgroup of triangular matrices in G. If μ_1 and μ_2 are two characters of $F^\times$ we may consider the induced representation $\xi = \mathrm{Ind}(G, B;\ \mu_1, \mu_2)$. There is a nonzero linear form λ on the space V of ξ such that

$$\lambda\left[\xi\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} f\right] = \psi(x)\lambda(f), \qquad f \in V.$$

The map which sends f to the function W, defined by

$$W(g) = \lambda[\xi(g)f], \tag{1}$$

is clearly bijective if ξ is irreducible, that is, if $\mu_1 \cdot \mu_2^{-1} \neq \alpha_F^{\pm 1}$ (we denote by α_F or α the module of F). If $\mu_1 \cdot \mu_2^{-1} = \alpha^{-1}$, the kernel of the map is one dimensional. If $\mu_1 \cdot \mu_2^{-1} = \alpha$ the map has trivial kernel. We recall the proof. Suppose more generally that $\mu_1 \cdot \mu_2^{-1} = \chi\alpha^u$ with $\chi\bar{\chi} = 1$ and $0 < u$. Then we may choose λ in such a way that

$$W\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix} = \hat{H}(-a)\mu_2(a)\,|a|^{1/2}, \qquad \hat{H}(a) = \int H(x)\psi(xa)\,dx,$$

where H is the element of $L^1(F)$ defined by

$$H(x) = f\left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}\right].$$

From the Fourier inversion formula, $W|B$ implies $H = 0$ and then, by continuity, $f = 0$. Thus we have proved the injectivity of the map $f \mapsto W$ and even the fact that the W's are determined by their restriction to B.

(1.2) In this paper we extend this result (and its proof) to the group $G_r = \mathrm{GL}(r, F)$, $r \geq 2$. In a precise way, let Q be the upper standard