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WITT KERNELS OF FUNCTION FIELD EXTENSIONS

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Let F be a field of characteristic not 2. For a non-hyperbolic quadratic form q of dimension at least 2, let $F(q)$ denote the function field of the projective variety $q = 0$. We consider the problem, explicitly raised as problem D by Lam, of determining the kernel of induced map of Witt rings $WF \to WF(q)$. This kernel is the Witt kernel of the field extension and is denoted by $W(F(q)/F)$. The basic tool is a comparison of $W(F(q \perp \langle x \rangle)/F)$ and $W(F(q)/F)$. The Witt kernels $W(F(q)/F)$ where q has small dimension or F has small Hasse number are determined. Applications are made to the question of when a conservative form is embeddable.

In the case q is a Pfister form, the function fields $F(q)$ have been widely used (e.g. the Arason-Pfister Hauptsatz). Central to the applications is that the Witt kernel $W(F(q)/F)$ is qWF for Pfister forms q. Elman, Lam and Wadsworth have considered function fields of several Pfister forms ρ_i, (cf. [**8**]). Again the basic problem is computing the Witt kernel $W(F(\rho_1, \rho_2, \ldots, \rho_r)/F)$ and showing it is a Pfister ideal.

Here also the emphasis is on finding conditions to insure Witt kernels are generated by Pfister forms. In the first section the comparison of $W(F(\varphi \perp \langle x \rangle)/F)$ and $W(F(\varphi)/F)$ is made and this is applied in the second section to forms of small dimension. For example, we show the Witt kernel $W(F(\varphi)/F)$ is a strong Pfister ideal if φ has dimension ≤ 5 and a Pfister ideal if dimension 6. This is used to improve several results of Gentile and Shaprio (in [**12**]) on their question of when $W(F(\varphi)/F)$ contains a non-zero Pfister form.

The last section treats fields F of finite Hasse number. It is shown that all Witt kernels of function fields are strong Pfister ideals if $\tilde{u}(F) \leq 8$. And the Witt kernels $W(F(\varphi)/F)$ are essentially computed for any form φ over F with $\tilde{u}(F) \leq 32$. Examples of fields with Hasse number ≤ 8 are C_3 fields, global and local fields, and finite fields.

The notation and terminology used are basically those of [**15**]. Isometry of forms α and β are denoted by $\alpha \simeq \beta$, while equality in the Witt ring is written $\alpha = \beta$. The uniquely determined maximal anisotropic subform α of a form β is termed the kernel of β and written as $\alpha = \ker(\beta)$. If $x\alpha \simeq \beta$ for some $x \in \dot{F}$, we say α and β are similar. The u-invariant used in the