PACIFIC JOURNAL OF MATHEMATICS
Vol 109, No 1, 1983

FORMAL REDUCTION THEORY OF MEROMORPHIC DIFFERENTIAL EQUATIONS: A GROUP THEORETIC VIEW

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One of the main goals of this paper is to develop an algorithm for reducing the first order (singular) system of differential equations:

$$(\dagger)\qquad \frac{df}{dz} = A(z)f$$

to a Turrittin-Levelt canonical form. Here $A(z) = z^r A_r + z^{r+1}A_{r+1} + \cdots$, $r < -1$ **and** $A_{r+m} \in \mathfrak{gl}(n; \mathbf{C})$ $m \geq 0$. **The reduction of (†) to a canonical form is implemented by the natural gauge adjoint action of** $\mathrm{GL}(n; \overline{\mathfrak{F}})$ **where** $\overline{\mathfrak{F}}$ **is the algebraic closure of the field of formal Laurent series about 0 with at most a finite pole at 0. For example, it is shown that the irregular part of the canonical form (†) is determined by** A_{r+m}, $0 \leq m < n(|r| - 1)$. **The proofs utilize group theoretic techniques as well as the method of Galois descent. In particular almost all of the results generalize to the case where** $\mathrm{GL}(n)$ **and** $\mathfrak{gl}(n)$ **are replaced by an arbitrary affine algebraic group** G **over C and its Lie algebra** $\mathfrak{g}$.

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References

0. Introduction and summary.

0.1. This paper presents a formal classification of meromorphic linear ordinary differential equations from a group theoretic point of view. Let