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ON UNITS OF PURE QUARTIC NUMBER FIELDS

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Let $K = Q(\sqrt[4]{D^4 \pm d})$ be a pure quartic number field, where D and d are natural numbers such that d divides D^3 and d is fourth power free. Then $\varepsilon = \pm(\sqrt[4]{D^4 \pm d} + D)/(\sqrt[4]{D^4 \pm d} - D)$ is a unit of K whose relative norm to the quadratic subfield of K is 1. We consider the condition for ε to be a member of a system of fundamental units of K.

1. Introduction. There have been many investigations concerning units of pure extensions of the rational number field of degree $n > 2$ generated by $\sqrt[n]{D^n \pm d}$, where D and d are natural numbers satisfying certain conditions ([**2**], [**4**], [**7**], [**9**], etc.). In general, suppose d divides D^{n-1} or, if n is a power of a prime number p, d divides pD^{n-1}. Then the numbers

$$\varepsilon_k = \frac{\omega^k - D^k}{(\omega - D)^k}, \qquad \omega = \sqrt[n]{D^n \pm d},$$

where k runs over all the divisors of n except 1, are units and, moreover, independent in the real algebraic number field generated by ω [**1**], [**2**], [**4**]. (The proof of independence of the ε_k's given by Halter-Koch and Stender [**4**] is incomplete. But the proof can be corrected by a slight modification.) When $n = 3$, 4 or 6, the number of such divisors is equal to the rank of the unit group of the field $Q(\omega)$, where Q denotes the rational number field. In this paper we shall treat these units in the case $n = 4$.

The following result is established by Stender [**8**], [**9**]:

Let D and d be two natural numbers such that $d \mid 2D^3$, and put $A = D^4 \pm d$ and $\omega = \sqrt[4]{A}$. Suppose that d is fourth power free and A/d or $2A/d$ is square free, according as $d \mid D^3$ or $d \mid 2D^3$. Then

$$\varepsilon_2 = \pm\frac{\omega + D}{\omega - D} \quad \text{and} \quad \varepsilon_4 = \begin{cases} \dfrac{d}{(\omega - D)^4} & \text{if } d \text{ is not a square,} \\[2ex] \dfrac{\sqrt{d}}{(\omega - D)^2} & \text{if } d \text{ is a square and } d \neq 1, \\[2ex] \pm\dfrac{1}{\omega - D} & \text{if } d = 1 \end{cases}$$