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ON GAMELIN CONSTANTS

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The purpose of this paper is to show that the corona theorem with bounds is valid for any finite bordered Riemann surface. As an application of it we construct an example of Riemann surfaces of infinite genus for which the corona theorem holds. The example can be chosen either from or not from the class of surfaces of Parreau-Widom type.

1. Introduction. Let R be a Riemann surface and $H^\infty(R)$ be the algebra of bounded analytic functions on R. Given a Riemann surface R, a natural number n and a positive number δ, we denote by $C_R(n, \delta)$ the infimum among constants C having the following property: For any $f_1, \ldots, f_n \in H^\infty(R)$ with $1 \geq \max_j |f_j| \geq \delta$ on R, there exist $g_1, \ldots, g_n \in H^\infty(R)$ such that $\sum_j f_j g_j = 1$ on R and $|g_j| \leq C$ on R ($j = 1, \ldots, n$). If there exist no such constants, then we define $C_R(n, \delta) = \infty$. We call $C_K(n, \delta)$ the *Gamelin constant* for the triple (R, n, δ). If $C_R(n, \delta) < \infty$ for every n and $\delta > 0$, then we say that *the Gamelin constant of R is finite*.

Gamelin [3] proved that the Gamelin constant of any finitely connected planar domain R is finite in such a way that $C_R(n, \delta)$ is dominated by a constant $C_m(n, \delta)$ depending only on n, δ and the number m of boundary components of R. The primary purpose of this paper is to prove the following.

THEOREM 1. *The Gamelin constant of any Riemann surface which is the interior of any finite bordered Riemann surface is finite.*

We raise the question of whether the constants can be chosen to depend only on the genus or rather on the Euler characteristic of the surface.

We denote the maximal ideal space of $H^\infty(R)$ by $\mathfrak{M}(R)$. We set $\tau(R) = \{$the homomorphisms "evaluation at p": $p \in R\}$. If $H^\infty(R)$ separates the points of R, we identify $\tau(R)$ with R. When $\tau(R)$ is dense in $\mathfrak{M}(R)$, we say that the *corona theorem* holds for R. The set $\tau(R)$ is dense in $\mathfrak{M}(R)$ if and only if the following property holds: For each n and $\delta > 0$, given $f_1, \ldots, f_n \in H^\infty(R)$ such that $\max |f_j| \geq \delta$ on R, there exist $g_1, \ldots, g_n \in H^\infty(R)$ such that $\sum_j f_j g_j = 1$ on R. Therefore if the Gamelin constant of R is finite, then the corona theorem holds for R. It is well