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WEAK COMPACTNESS IN SPACES OF BOCHNER INTEGRABLE FUNCTIONS AND THE RADON-NIKODYM PROPERTY

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We characterize Banach spaces E such that E and E^* have the Radon-Nikodym property in terms of relatively weakly compact sets of $L^1[\lambda, E]$.

Introduction. It is well known [**1**] that if $(\Omega, \Sigma, \lambda)$ is a finite measure space and E is a Banach space, then a relatively weakly compact subset K of $L^1[\lambda, E]$ is *bounded*, *uniformly integrable* and for every $B \in \Sigma$, the set $\{\int_B f\,d\lambda, f \in K\}$ is *relatively weakly compact* in E. Moreover, it was shown in [**1**] that if the Banach space E and its dual E^* have the Radon-Nikodym property, then relatively weakly compact subsets of $L^1[\lambda, E]$ are completely characterized by the above three conditions. A question that arises naturally is the following: Are the conditions on E and E^* to have the Radon-Nikodym property necessary in order that relatively weakly compact subsets of $L^1[\lambda, E]$ be exactly those bounded, uniformly integrable subsets K such that for any $B \in \Sigma$, the set $\{\int_B f\,d\lambda,\ f \in K\}$ is relatively weakly compact in E? In [**1**], it was shown that the condition on E to have the Radon-Nikodym property is indeed necessary. The object of this paper is to show that the condition on E^* to have the Radon-Nikodym property is also necessary. This gives a new characterization of Banach spaces E such that E and E^* have the Radon-Nikodym property. We also study bounded linear operators T between Banach spaces such that T and its adjoint T^* are strong Radon-Nikodym operators.

Definitions and Preliminaries.

DEFINITION 1. A closed bounded convex subset C of a Banach space E is a *Radon-Nikodym* (R.N.P) *set* if for every finite measure space $(\Omega, \Sigma, \lambda)$ and any vector measure $G: \Sigma \to E$ such that the set $\{G(B)/\lambda(B), B \in \Sigma, \lambda(B) > 0\}$ is contained in C, there exists a Bochner integrable Radon-Nikodym derivative $f: \Omega \to C$ such that $G(B) = \int_B f\,d\lambda$, for every $B \in \Sigma$.

For more on (R.N.P) sets see [**3**] and [**4**].