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ATOROIDAL, IRREDUCIBLE 3-MANIFOLDS AND 3-FOLD BRANCHED COVERINGS OF S^3

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Suppose M is a closed orientable 3-manifold. Then H. Hilden et al. proved that M is a 3-fold branched covering of S^3 branched over a fibered knot. In this paper we prove that, if M is irreducible and atoroidal, then M is either a 3-fold branched covering of S^3 branched over a simple, fibered knot, or a 2-fold branched covering of a closed orientable 3-manifold whose Heegaard genus is at most one.

Hilden [4], Hirsch [5] and Montesinos [11] proved independently that a closed, connected and orientable 3-manifold M is a 3-fold irregular branched covering of S^3 branched over a knot K. Further, it is known that K may be chosen to be a fibered knot. We do not know a reference for this refinement, which we need for our main theorem, so we give in §1 a sketch of the proof, shown to us by Hilden. Our main result is:

THEOREM. *Let M be a closed, connected and orientable* 3*-manifold. Suppose M is atoroidal and irreducible. Then at least one of the following holds.*

(i) *M is a* 3*-fold* (*cyclic or irregular*) *branched covering of S^3 branched over a simple, fibered knot.*

(ii) *There exist a closed, connected and orientable* 3*-manifold N whose Heegaard genus is at most one and a simple link L in N such that M is a* 2*-fold branched covering of N branched over L.*

Here M *atoroidal* means M contains no embedded incompressible torus. As is well known, classifying closed orientable 3-manifolds essentially reduces to the case of atoroidal irreducible 3-manifolds, by the Unique Prime Decomposition Theorem [9] and the Torus Decomposition Theorem [6], [7].

Recently Thurston announced that, if an atoroidal and irreducible 3-manifold M is a regular (in particular cyclic) branched covering of a closed, orientable 3-manifold, then M has a *geometric structure* (i.e. M admits a complete riemannian metric in which any two points have isometric neighborhoods). By this result and our Theorem, if M is a closed, orientable 3-manifold which is atoroidal and irreducible, then M