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INVERSE SPECTRAL PROBLEMS FOR CERTAIN DIFFERENTIAL OPERATORS

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Let L_0 be a given differential operator with spectral matrix (ρ_{ij}^0). There is a concept of "closeness to (ρ_{ij}^0)" such that for every positive matrix measure (ρ_{ij}) which is "close to (ρ_{ij}^0)" there exists some differential operator L for which (ρ_{ij}) is a spectral matrix and there exists a potentially computational technique by which L may be constructed from (ρ_{ij}) and (ρ_{ij}^0). The formulation of the "closeness to (ρ_{ij}^0)" concept and the presentation of the techniques by which L may be constructed from (ρ_{ij}) and (ρ_{ij}^0) are referred to as the local inverse spectral problem, which is the subject of this paper.

Introduction. Sahnovič [**6**] has presented a formulation of the local inverse spectral problem but he defines "closeness to (ρ_{ij}^0)" in a manner that is too restrictive and excludes many solvable cases. For example many problems in the second order case, which had previously been solved by Gelfand and Levitan [**2**], do not meet the "closeness" criterion of Sahnovič. On the other hand his presentation omits some necessary technical conditions that, despite their awkward appearance, must be assumed in case $2n > 2$.

The present article gives the least restrictive conditions possible, which in the second order case coincide with the conditions given by Gelfand and Levitan.

The above changes require modifications of the technique by which the differential operator L is constructed from (ρ_{ij}) and (ρ_{ij}^0).

CHAPTER 1
TECHNICAL PRELIMINARIES

1. Orientation. Let l be the differential expression defined by

$$lu = (-1)^n u^{(2n)} + (-1)^{n-1}\left(p_1 u^{(n-1)}\right)^{(n-1)} + \cdots + p_n u,$$

where the coefficients $p_k(x)$ are real valued functions on $[0, \infty)$ that are locally integrable. Any formally selfadjoint differential expression defined on the positive real axis that is regular at zero and has sufficiently differentiable real coefficients can be put into this form if $p_0 \equiv 1$. On the