

## SOME EXAMPLES OF RIEMANNIAN ALMOST-PRODUCT MANIFOLDS

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Foliations with bundle-like metrics, conformal, minimal and totally geodesic foliations and minimal and geodesic plane fields have been subject to recent study. A. M. Naveira has fitted all these classes into a general scheme and has gotten thirty six classes of riemannian almost-product manifolds. In this paper we give strict examples of these classes, showing that none of them is vacuous, and that the inclusion relations among them are strict. The basic riemannian manifolds for the construction of these examples are submanifolds of  $C^{n+1}$  and  $H^{n+1}$  ( $C$  = complex field,  $H$  = quaternion field), and we use the canonical complex structures on these vector spaces. Perhaps the most interesting examples are those of minimal foliations which are not totally geodesic foliations.

**1. Introduction.** Some classes of riemannian almost-product manifolds have been extensively studied in the literature. In the beginning, most of the works on this subject dealt with foliations with bundle-like metrics ([6], [17], [15]). Recently, the study of other types of riemannian almost-product manifolds has been initiated: conformal foliations ([14], [23], [11], [12]), minimal and geodesic plane fields ([18], [24]), minimal foliations ([5], [16], [20], [21]), totally geodesic foliations ([2], [9]). Hsu [8], by analogy with the almost-hermitian manifolds, defined some types of riemannian almost-product manifolds in terms of the symmetries of the tensor  $\nabla P$ , where  $\nabla$  is the Levi-Civita connection and  $P$  is the almost-product tensor ( $P^2 = \text{identity}$ ). Naveira [13] has fitted all these classes into a general scheme. This has been accomplished by means of a detailed study of a representation of the pertinent group  $O(p) \times O(q)$  on a certain vector space. With this method he has gotten thirty six classes of riemannian almost-product manifolds, and has also given a geometric interpretation of some. Carreras [1], along the study of the natural functions on riemannian almost-product manifolds, has shown that this general scheme is, in a reasonable sense, complete. Gil-Medrano [3] and Montesinos [11] have completed the geometric interpretation.

In this note we give concrete examples of some classes. By using them and the behaviour of the classes under a conformal transformation of the metric studied in [3], we obtain examples of the thirty six classes, which shows that the inclusion relations among the classes are strict.