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ASYMPTOTIC BEHAVIOR OF A PERTURBED NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATION RELATED TO THE SOLUTION OF THE UNPERTURBED LINEAR SYSTEM

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In this paper we consider the problem of the relative asymptotic equivalence of the solutions of the systems

$$\frac{d}{dt} Dy_t = L(y_t) \tag{1}$$

and

$$\frac{d}{dt}\left[Dx_t - G(t, x_t)\right] = L(x_t) + f(t, x_t), \tag{2}$$

where (1) is a linear system of neutral functional differential equations. The main theorem gives conditions under which the following result is verified. Given a solution y_t of (1) there exists a solution x_t of (2) such that

$$\lim \frac{\|x_t - y_t\|}{\|y_t\|} = 0. \tag{*}$$

The converse of this result, namely given a solution x_t of (2) there is a solution y_t of (1) such that (*) is satisfied is partially proved. A counter-example is given to show that the converse result is not true in general.

0. Introduction. The study of asymptotic behavior of differential equations is very important to the understanding of the qualitative behavior of the solutions of an ordinary differential equation. Several mathematicians, including N. Levinson, H. Weil, P. Hartman, R. Bellman, K. Cooke, J. Hale, L. Cesari, and others, have done a great deal of work in this area. The theory of functional differential equations is relatively new, having evolved mainly in the last twenty years, and not many papers have appeared on asymptotic behavior of functional differential equations. One early paper was published by Bellman and Cooke in 1959 [**1**], followed by several others that consider a nonlinear delay equation as a perturbation of an ordinary differential equation; see, for example, Cooke [**3**]. Although this point of view is important, in some cases a better approach is to consider a nonlinear functional differential equation because the linearized equation is still a linear functional differential equation, and because the difficulties involved in the solution of the problem spring