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BIJECTIVELY RELATED SPACES I: MANIFOLDS

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The following equivalence relation is introduced: Two (Hausdorff) spaces X and Y are *bijectively related* if there exist continuous bijections $f: X \to Y$ and $g: X \to Y$. This first paper considers the case in which X and Y are connected manifolds. If either f or g is not a homeomorphism, then each space is necessarily non-reversible and hence this study produces more knowledge of such spaces. The chief results here are the existence theorem (Theorem 2) and, perhaps, Corollary 12, which states that a simply-connected manifold having only compact boundary components is reversible.

This is a continuation of a study of continuous bijections following the work of Rajagopalan and Wilansky **[5]**, Petty **[4]**, and Doyle and Hocking **[2, 3]**. We introduce here the following equivalence relation among topological spaces:

DEFINITION. Two spaces X and Y are *bijectively related* if there exist continuous bijections $f: X \to Y$ and $g: Y \to X$. Each space is then a *bijective relative* of the other, the maps f and g are *relating bijections* and, to be brief, we say that "$[X, Y, f, g]$ holds". We denote by $B(X)$ the equivalence class of all spaces (in the category under study) which are bijectively related to X.

1. Preliminaries. Throughout this study spaces will be assumed to be Hausdorff (at least). With this assumption we surely have $B(X) = \{X\}$ if X is compact. To provide a more general result in this direction, recall that the space X is said to be *reversible* **[5]** if the only continuous self-bijections $f: X \to X$ are the homeomorphisms. If X is reversible and if $[X, Y, f, g]$ holds, then $g \circ f: X \to X$ must be a homeomorphism. Then $f^{-1} = (g \circ f)^{-1} \circ g$ is continuous, so f is a homeomorphism. Thus $B(X) = \{X\}$ whenever X is reversible. However the condition $B(X) = \{X\}$ does not characterize reversible spaces, as we see next.

THEOREM 1. *Among metric spaces the rationals Q constitute a non-reversible space for which $B(Q) = \{Q\}$.*