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A MARCINKIEWICZ CRITERION FOR L^p-MULTIPLIERS

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Suppose m is a bounded measurable function on the n-dimensional Euclidean space $\mathbf{R}^n$. Define a linear operator T_m by $(T_m f)^\wedge = mf^\wedge$, where $f \in L^2 \cap L^p(\mathbf{R}^n)$, $1 \leq p \leq \infty$, and $f^\wedge$ denotes the Fourier transform of f:

$$f^\wedge(\xi) := \int f(x) e^{-ix\xi}\, dx \qquad \left(x\xi := \sum_{j=1}^{n} x_j \xi_j \right).$$

(We omit the domain of integration if it is the whole $\mathbf{R}^n$.) If T_m is bounded from $L^p(\mathbf{R}^n)$ to $L^p(\mathbf{R}^n)$, then m is called an L^p-(Fourier) multiplier, denoted $m \in M_p(\mathbf{R}^n)$. The norm of m coincides with the operator norm of T_m.

THEOREM 1. ***Let** m **and** m' **be locally absolutely continuous on** $(0, \infty)$ **and***

$$B := \|m\|_\infty + \sup_{j \in Z} \int_{2^j}^{2^{j+1}} r |m''(r)|\, dr < \infty.$$

***Then** $m(|\xi|) \in M_p(\mathbf{R}^n)$ **for all** p **with** $1 \leq 2n/(n+3) < p < 2n/(n-3) \leq \infty$; **in particular**, $\|m\|_{M_p(\mathbf{R}^n)} \leq cB$ **with** c **independent of** m.*

1. To prove Theorem 1 we need a result stated in Theorem 2 about the following Littlewood-Paley function:

$$g_\lambda(f)(x) = \left(\int_0^\infty | S_t^{\lambda+1}(f; x) - S_t^\lambda(f; x)|^2 u(t) \frac{dt}{t} \right)^{1/2}, \tag{1.1}$$

where

$$S_t^\lambda(f; x) = \int \left(1 - \frac{|\xi|^2}{t^2} \right)_+^\lambda f^\wedge(\xi) e^{i\xi x}\, d\xi \qquad (r_+ = \max(0, r))$$

denotes the Bochner-Riesz means of f of order λ, u is a nonnegative measurable function on $(0, \infty)$ satisfying

$$t \leq R(t) = \int_0^t u(s)\, ds \leq ct, \qquad t > 0, \tag{1.2}$$

and f belongs to S, the space of all infinitely differentiable rapidly decreasing functions on $\mathbf{R}^n$.