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AN INTERPOLATION THEOREM AND ITS APPLICATIONS TO POSITIVE OPERATORS

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We answer a special case of a problem of Z. Ditzian. The obtained estimate for the Peetre K functional is applied to the characterization of functions for which $\|T_n f - f\| = O(n^{-\alpha})$ $(0 < \alpha < 1)$, where T_n is either the Bernstein, Szász-Mirakjan or Baskakov operator or their Kantorovich-invariant and $\|\cdot\|$ denotes either the L^p $(p \geq 1)$ or the supremum norm.

1. Let (a, b) be an interval, $B = L^p(a, b)$ $(1 \leq p < \infty)$ or $B = C[a, b]$, φ a non-negative function on (a, b) and $r \geq 1$ an integer. Z. Ditzian [**6**] estimated the Peetre K functional

$$K_r(t^r, f) = \inf_{g, g^{(r-1)} \text{abs. cont.}} \left(\|f - g\|_B + t^r \|\varphi^r g^{(r)}\|_B\right)$$

by norms of second order differences of f when φ had certain regularity conditions. In connection with this he raised the problem if in the case $(a, b) = (0, 1)$, $B = L^p(0, 1)$ $(1 \leq p < \infty)$, $\varphi(x) = x^\alpha$ $(\alpha > 0)$, $f \in B$, $(\text{support} f) \subseteq (0, 3/4)$, the estimate

$$D_1 \omega_{2r}^{**}(f, t) \leq K_{2r}(t^{2r}, f) \leq D_2 \omega_{2r}^{**}(f, t) \tag{1.1}$$

holds, where

$$\omega_{2r}^{**}(f, t) = \sup_{\eta \leq t} \left\{ \int_{(r\eta)^{1/(1-\alpha)}}^{1-C} |\Delta_{\eta x^\alpha}^{2r} f(x)|^p \, dx \right\}^{1/p}$$
$$+ \sup_{\eta \leq t^{1/(1-\alpha)}} \left\{ \int_{r\eta} |\Delta_\eta^{2r} f(x)|^p \, dx \right\}^{1/p} \quad \text{for } 0 < \alpha < 1,$$

$$\omega_{2r}^{**}(f, t) = \sup_{\eta \leq t} \left\{ \int_{(r\eta)^{1/(1-\alpha)}}^{1-C} |\Delta_{\eta x^\alpha}^{2r} f(x)|^p \, dx \right\}^{1/p} \quad \text{for } \alpha \geq 1,$$

and

$$\Delta_h^2 f(x) = f(x - h) - 2f(x) + f(x + h).$$