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ITERATED AVERAGING FOR PERIODIC SYSTEMS WITH HIDDEN MULTI-SCALE SLOW TIMES

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General asymptotic methods on various time scales are developed for periodic systems of ordinary differential equations in order to treat global motion in multi-oscillatory systems. Moreover, we show that bifurcations of an attractive and essentially nonperiodic nature can arise in systems that also possess several (often unstable) Hopf bifurcations. Such attractor bifurcations frequently dominate the long term system behavior. In addition, the methods here can be used to determine the flow on a center manifold in cases where center manifold theory indicates an instability at the origin of that manifold and little else about the flow. Finally, various examples of mixed scale motion are treated.

1. Introduction. A number of methods have been developed for assessing the behavior of systems of ordinary differential equations. Hopf bifurcation theory as illustrated in J. Marsden and M. McCracken [7] and in A. Poore [13] is a well-known example. And additional results in the treatment of bifurcations have been obtained by K. Landman and S. Rosenblatt [5] and by W. Langford [6]. However, none of the foregoing works comes to grips with bifurcations that either possess an exceptionally large least period or else possess no period at all. This limitation becomes particularly significant for systems possessing two or more characteristic oscillatory frequencies. For instance, considering Example 1 in §2 of this paper, Hopf bifurcation theory (see [7, p. 96]) shows the existence of two different periodic bifurcations, the first having a period approximately equal to 2π and the second a period approximately equal to $2\pi/\alpha$ (where 1 and α are the angular frequencies of the system). However, using S. Persek [11], both bifurcations can be shown to be unstable, and thus neither of them characterizes the long term motion of the system. Therefore, in Example 1, we arrange to locate still another bifurcation solution, and as is typical in such cases, this solution is either nonperiodic or, if periodic, has no period smaller in magnitude than order $1/\varepsilon^3$ (where $\varepsilon > 0$ is arbitrary and small). Now the fact that no period of this solution can be as small as 2π or $2\pi/\alpha$ rules out the posibility of the solution being a Hopf bifurcation (or even of its being discoverable by that approach). Nevertheless, this solution is a perfectly well-behaved bifurcation which, because of its general characteristics as an attractor, describes the long term motion of the system. In fact, the system will drift away from the