PACIFIC JOURNAL OF MATHEMATICS
Vol. 112, No. 1, 1984

# THE SPACE OF EXTENDED ORTHOMORPHISMS IN A RIESZ SPACE

B. DE PAGTER

**We study the space $\mathrm{Orth}^{\infty}(L)$ of extended orthomorphisms in an Archimedean Riesz space $L$ and its analogies with the complete ring of quotients of a commutative ring with unit element. It is shown that for any uniformly complete $f$-algebra $A$ with unit element, $\mathrm{Orth}^{\infty}(A)$ is isomorphic with the complete ring of quotients of $A$. Furthermore, it is proved that for any uniformly complete Riesz space $L$ the space $\mathrm{Orth}^{\infty}(L)$ is isomorphic to the lateral completion of $L$. Finally, it is shown that for any uniformly complete Riesz space $L$ the ring $\mathrm{Orth}^{\infty}(L)$ is von Neumann regular.**

The main subject in this paper is the space $\mathrm{Orth}^{\infty}(L)$ of extended orthomorphisms in an Archimedean Riesz space $L$. By an extended orthomorphism we mean an order bounded linear mapping $\pi$ from an order dense ideal $D$ in $L$ into $L$ with the property that $\pi f \perp g$ for all $f \in D$ and $g \in L$ with $f \perp g$. As shown in [**10**], $\mathrm{Orth}^{\infty}(L)$ is an Archimedean $f$-algebra with unit element which is, in addition, laterally complete.

The definition of $\mathrm{Orth}^{\infty}(L)$ for an Archimedean Riesz space is in some sense analogous to the definition of the complete ring of quotients $Q(R)$ of a commutative ring $R$ with unit element (see [**8**], §2.3). A natural thing to do, therefore, is to compare these two objects for Archimedean $f$-algebras with unit element. In §2 of this paper it is proved that for any uniformly complete $f$-algebra $A$ with unit element, the algebras $\mathrm{Orth}^{\infty}(A)$ and $Q(A)$ are indeed isomorphic.

For any $f$-algebra $A = C(X)$, where $X$ is a completely regular Hausdorff space, the complete ring of quotients of $A$ is precisely the lateral completion $A^{\lambda}$ of $A$. So, by the above-mentioned result, in this case $\mathrm{Orth}^{\infty}(A)$ is the lateral completion of $A$. In §3 we study the relation between $\mathrm{Orth}^{\infty}(L)$ and the lateral completion $L^{\lambda}$ for an arbitrary Archimedean Riesz space, and it will be shown that $\mathrm{Orth}^{\infty}(L) = L^{\lambda}$ holds for uniformly complete Riesz spaces.

Another interesting property of the ring of quotients $Q(R)$ of a semiprime commutative ring $R$ with unit element is that $Q(R)$ is von Neumann regular. In the last section of this paper it will be shown that $\mathrm{Orth}^{\infty}(L)$ is a von Neumann regular $f$-algebra for any uniformly complete Riesz space $L$.