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A DESCRIPTION OF THE TOPOLOGY ON THE DUAL SPACE OF A NILPOTENT LIE GROUP

KENNETH I. JOY

The study of convergence of sequences of elements of the dual space $\hat{G}$, for a nilpotent Lie group G, is done by reducing the study to convergence of sequences of subgroup representation pairs, whose subgroup component has dimension less than the dimension of G. The main results are then applied to give a new proof to the fact that the Kirillov correspondence is a homeomorphism for nilpotent Lie groups.

1. Introduction. Let G be a real, connected, simply-connected nilpotent Lie group. By the dual space $\hat{G}$ of G, we mean the set of all equivalence classes of irreducible unitary representations equipped with the hull-Kernel topology. Kirillov [4] has shown that the elements of $\hat{G}$ are in one-to-one correspondence with orbits of real linear functionals on $\mathfrak{g}$ (the Lie algebra of G). The fact that this correspondence is actually a homeomorphism was conjectured by Kirillov and first proved by Brown [1].

In this paper we study the convergence of sequences $\{W_n\}$ in $\hat{G}$ by studying sequences of subgroup-representation pairs $\{(H_n, S_n)\}$, where H_n is a subgroup of G, $S_n \in \hat{H}_n$ and $\dim H_n < \dim G$. We develop necessary and sufficient conditions for the convergence of sequences in $\hat{G}$, in terms of convergence of associated sequences of subgroup-representation pairs in the subgroup-representation topology of Fell [3]. Then using the theorems of Fell, we give a new proof of the Kirillov conjecture that avoids the use of the free nilpotent Lie algebras of Brown.

2. Preliminaries. Let G be a simply connected nilpotent Lie group. We will be primarily interested in the case when G is nonabelian. For if G were abelian, then $G \cong \mathbf{R}^n$ for some n, and the irreducible representations of G are characters. In fact, $\hat{\mathbf{R}}^n \cong \mathbf{R}^n$. Therefore, the case when G is abelian is essentially trivial and, unless otherwise noted, we will assume our groups are nonabelian.

We introduce the following conventions in notation:

(1) $\mathcal{K}(G)$ will denote the set of all closed subgroups of G, equipped with the compact-open topology

(2) $\mathcal{Q}(G)$ will denote the set of all subgroup-representation pairs of G, equipped with the subgroup-representation pair topology of Fell [3]