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INEQUALITIES FOR EIGENVALUES OF THE BIHARMONIC OPERATOR

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Let D be a bounded domain in R^m with smooth boundary. The first $n+1$ eigenvalues for the problem

$$\Delta^2 u - \mu u = 0 \quad \text{in } D, \qquad u = \frac{\partial u}{\partial n} = 0 \quad \text{on } \partial D$$

satisfy the inequality

$$\sum_{i=1}^{n} \frac{\sqrt{\mu_i}}{\mu_{n+1} - \mu_i} \geq \frac{m^2 n^{3/2}}{8(m+2)} \left(\sum_{i=1}^{n} \mu_i \right)^{-1/2}$$

For the first two eigenvalues we have the stronger bound

$$\mu_2 \leq 7.103\, \mu_1 \quad (\text{in } R^2),$$
$$\mu_2 \leq 4.792\, \mu_1 \quad (\text{in } R^3).$$

The first two eigenvalues for the problem

$$\Delta^2 u + \nu \Delta u = 0 \quad \text{in } D, \qquad u = \frac{\partial u}{\partial n} = 0 \quad \text{on } \partial D$$

satisfy the inequality

$$\nu_2 \leq 2.5\ \nu_1 \quad (\text{in } R^2),$$
$$\nu_2 \leq 2.12\, \nu_1 \quad (\text{in } R^3).$$

Introduction. Let D be a bounded domain in R^m, $m \geq 2$, with smooth boundary ∂D. For the case $m = 2$, Payne, Polya and Weinberger [**6**] obtained upper estimates, independent of the domain, for eigenvalues of the three well-known eigenvalue problems:

$$\Delta u + \lambda u = 0 \quad \text{in } D, \quad u = 0 \quad \text{on } \partial D, \tag{1}$$

$$\Delta^2 u - \mu u = 0 \quad \text{in } D, \quad u = \frac{\partial u}{\partial n} = 0 \quad \text{on } \partial D, \tag{2}$$

$$\Delta^2 u + \nu \Delta u = 0 \quad \text{in } D, \quad u = \frac{\partial u}{\partial n} = 0 \quad \text{on } \partial D. \tag{3}$$

Let

$$0 < \lambda_1 < \lambda_2 \leq \lambda_3 \leq \cdots,$$
$$0 < \mu_1 \leq \mu_2 \leq \mu_3 \leq \cdots,$$
$$0 < \nu_1 \leq \nu_2 \leq \nu_3 \leq \cdots$$