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DE RHAM THEOREM WITH CUBICAL FORMS

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With a simplicial complex X there is associated a commutative differential graded algebra of polynomial differential forms $T(X)$ together with a filtration $T^{*,q}(X) \subset T^{*,q+1}(X)$ in each degree $*$. $T^{*,q}(X)$ is a differential graded module over the subring of the rationals $Z[\frac{1}{2},\dots,\frac{1}{q}]$. The deRham theorem for such a complex $T(X)$ is proved. We have demonstrated elsewhere that the refined deRham complex $T(X)$ makes it possible to substantially refine most of the results of the rational homotopy theory. In particular we defined the homotopy category of C-N spaces which is equivalent to an algebraic homotopy category of $(N-1)$ connected free commutative differential graded algebras over the integers, satisfying a simple algebraic condition on cohomology.

1. Introduction. As a refinement of the deRham complex of rational differential forms, $A^*(X; Q)$ on a simplicial complex, Cartan [**1**] and Miller [**11**] defined a filtration $A^{*,q}(X; Z)$ of $A^*(X; Q)$ such that the cohomology $H^p(A^{*,q}(X; Z))$ is isomorphic to the singular cohomology $H^p(X; Z)$ for $p \leq q$. In our study, [**2**], [**3**], of the relationship between the fundamental group of X and the filtered algebra $A^{*,*}(X; Z)$, the best results are obtained under the assumption that $H_1(X; Z)$ is both finitely generated and free. An effort to eliminate the freeness assumption lead us to the construction of a new commutative filtered algebra of forms, $T^{*,*}(X)$. $T^{*,*}(X)$ is an analogue of the filtered Cartan-Miller forms obtained by replacing simplices with cubes. Since each simplicial complex has a canonical subdivision into cubes, $T^{*,*}(X)$ can be viewed as a functor defined on simplicial complexes. For fixed q, $T^{*,q}(X)$ is a complex of Q_q modules where Q_q denotes the smallest subring of the rationals containing $1/p$ for each prime p, $p \leq q$, $Q_0 = Q_1 = Z$. The usual wedge product of forms induces a map $T^{*,q_1}(X) \otimes T^{*,q_2}(X) \to T^{*,q_1+q_2}(X)$. The main result of this paper is that integration of forms over cubes induces an isomorphism I: $H^p(T^{*,q}(X)) \to H^p(X; Q_q)$ for $q \geq 1$ and for all p. I is multiplicative in the sense that the diagram

$$\begin{array}{ccc} H^{p_1}(T^{*,q_1}(X)) \otimes H^{p_2}(T^{*,q_2}(X)) & \overset{\wedge}{\to} & H^{p_1+p_2}(T^{*,q_1+q_2}(X)) \\ I \otimes I \downarrow & & I \\ H^{p_1}(X; Q_{q_1}) \otimes H^{p_2}(X; Q_{q_2}) & \overset{\cup}{\to} & H^{p_1+p_2}(X; Q_{q_1+q_2}) \end{array}$$

commutes.