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COMPARISON THEOREMS FOR SECOND-ORDER OPERATOR-VALUED LINEAR DIFFERENTIAL EQUATIONS

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Let B be a Banach lattice with order continuous norm, $\mathfrak{L}(B)$ the algebra of bounded linear operators. Let B_+ denote the positive cone induced by the lattice structure of B, and $\mathfrak{L}_+(B)$ the corresponding positive cone in $\mathfrak{L}(B)$. We consider second-order operator-valued differential equations of the form $Y'' + Q(x)Y = 0$, where Q: $[a, +\infty) \to \mathfrak{L}(B)$ is continuous in the uniform topology and is such that $\int_x^\infty Q(t)\,dt \in \mathfrak{L}_+(B)$ for all $x \geq a$. Comparison theorems of Hille-Wintner type are obtained.

1. Introduction. Consider the second-order linear differential equation

$$Y'' + Q(t)Y = 0 \tag{1.1}$$

where Q: $[a, +\infty) \to \mathfrak{L}(B)$ is a continuous operator-valued function and $\mathfrak{L}(B)$ represents the Banach algebra of bounded linear operators T: $B \to B$, where B denotes a Banach space. By a solution Y of (1.1) we understand a function Y: $[a, \infty) \to \mathfrak{L}(B)$ which is twice continuously differentiable in the uniform operator topology and satisfying (1.1) for all $t \in [a, \infty)$. We refer to the text of Hille **[12]** for a discussion of the concepts of differentiation and integration of functions from $[a, \infty)$ to a Banach algebra and to **[15]** for basic results concerning differential equations in Banach spaces. We shall be interested in comparing solutions of (1.1) with solutions of a second equation

$$Y'' + Q_1(t)Y = 0 \tag{1.2}$$

with Q_1: $[a, \infty) \to \mathfrak{L}(B)$ continuous. A solution $Y = Y(t)$ of (1.1) (or (1.2)) is said to be *non-singular* at a point $t_0 \in [a, \infty)$ if it has a bounded inverse $Y^{-1}(t_0) \in \mathfrak{L}(B)$. If $Y(t)$ is non-singular for all $t \in [t_0, +\infty)$, some $t_0 \geq a$, then $Y = Y(t)$ is said to be a non-oscillatory solution of (1.1) on $[t_0, +\infty)$. Otherwise, $Y = Y(t)$ is said to be oscillatory on $[a, \infty)$. (Note that the inverse $Y^{-1}(t)$ of a non-oscillatory solution $Y(t)$ of (1.1) is continuously differentiable.)

Equations (1.1) and (1.2) have been the subject of numerous investigators (**[4]**–**[11]**, **[19]**, and the references therein) for the case that B is a